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Example 3 Analysis of a Qualitative Predictor Variable

In Chapter 8 hypothetical mail survey data were used to demonstrate the importance of assessing the model fit to test the validity of the assumptions made by a model. In the current example we will analyze real data from the Mail Survey Experiment (Magidson, 1994b) where the actual dollar payments tested were *$1* (the control), *$2*, *$3* and *$4*, the experiment being designed so that there is no correlation between X_1:PAYMENT and X_2:CALL. We will again use the model fit statistic to choose between two alternative models, one that treats PAYMENT as a quantitative predictor with equidistant category scores (Model A) and one that treats PAYMENT as a qualitative predictor (Model B).

The data from the Mail Survey Experiment is summarized in Table 9-1 (included in file ex3.*). The observed rate of returning the survey is given for each joint category formed by the joint predictor X=CALL x PAYMENT, in percentages, odds and logit units. For example, among persons who received both a $1 payment and a reminder call, 55.2% responded. The observed odds in favor of response for this group is 55.2% / (100% - 55.2%) = 1.23 and the logit = ln(1.23) = 0.21.

		RETURN		Likelihood of Response		
CALL	**PAYMENT**	**Yes**	**No**	**Percentage**	**Odds**	**Logit**
Yes (1)	**$1 (1)**	2,407	1,954	55.2 %	1.23	0.21
	$2 (2)	1,265	881	58.9 %	1.44	0.36
	$3 (3)	1,340	809	62.4 %	1.66	0.50
	$4 (4)	1,306	779	62.6 %	1.68	0.52
No (0)	**$1 (1)**	2,133	2,176	49.5 %	0.98	-0.02
	$2 (2)	1,156	942	55.1 %	1.23	0.20
	$3 (3)	1,262	897	58.5 %	1.41	0.34
	$4 (4)	1,248	839	59.8 %	1.49	0.40

Table 9-1
Mail Survey Experiment with Descriptive Statistics

Model A, which treats PAYMENT as a *quantitative* predictor with equidistant category scores, was estimated using the **Uniform** scoring option. **Uniform** scores are equidistant scores that have a low score of 0 and a high score of 1.

The overall results for Model A are summarized in Figure 9-1 and the partial Effects plot for PAYMENT is displayed in Figure 9-2.

Figure 9-1
Model Summary Results of Model A which Specifies Equidistant Payment Scores

y: RETURN (Fixed)	0	1				
Y-scores	0.00	1.00				
Alpha	0.00	0.02				
		$L^2(Y)$	df	p-value	Beta	exp(Beta)
PAYMENT (Fixed)		119	1	1.1e-27	0.39	1.48
CALL (Fixed)		42.6	1	6.7e-11	0.18	1.20
Association Summary		L^2	df	p-value	R^2	
Explained by Model		161	2	1.2e-35	0.00749	
Residual		9.45	5	0.093		

Figure 9-2
Model A: PAYMENT is treated as Fixed with $1 as the reference

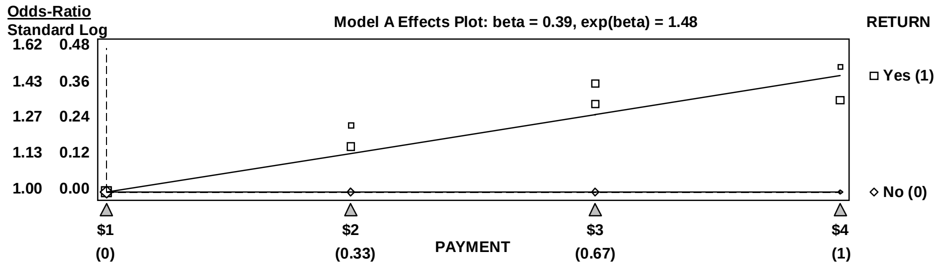


Figure 9-2 shows that persons receiving a $2 payment and for whom CALL = Yes(No) are estimated to be exp(0.33*$\hat{\beta}_1$) = 1.14 times as likely to return the survey than persons receiving a $1 payment and for whom CALL = Yes(No). Similarly, given CALL = Yes(No), the effect of a $3 payment is estimated to be exp(0.67*$\hat{\beta}_1$*) = 1.30 and a $4 payment is exp(*$\hat{\beta}_1$*) = 1.48 times as likely to return the survey than persons receiving a $1 payment (who were exposed to the same calling option). Uniform scores are displayed in parenthesis.*

Model B, which treats PAYMENT as a *qualitative* predictor, was estimated using the **Free** scaling option for the PAYMENT variable. In order that the category scores estimated for any **Free** scale variable such as PAYMENT be identifiable, two standardizing restrictions must be imposed to uniquely define a zero point and a unit. By default, like **Uniform** scores, GOLDMineR restricts the scores by fixing the lowest at 0 and highest at 1. The overall model results for Model B are summarized in Figure 9-3 and the corresponding Effects plot is displayed in Figure 9-4. The estimated PAYMENT scores, labeled X-score in Figure 9-3, are used to position the payment levels on the horizontal axis of the partial Effects plot.

Figure 9-3
Model Summary Results from Model B which specifies Free PAYMENT Scores (with $1 payment serving as the reference category)

y: RETURN (Fixed)	0	1
Y-scores	0.00	1.00
Alpha	0.00	0.00

	$L^2(Y)$	df	p-value	Beta	exp(Beta)
PAYMENT (Free)	126	3	3.9e-27	0.36	1.44
CALL (Fixed)	42.6	1	6.7e-11	0.18	1.20

Association Summary	L^2	df	p-value	R^2
Explained by Model	168	4	2.9e-35	0.00783
Residual	2.38	3	0.50	
Total	170	7	2.2e-33	

Category-Specific Parameter Estimates

PAYMENT (Free)	X-score	C-weights	Beta(k)	Std. Err.	Wald	p-value	exp(Beta)	Lower	Upper
1	0.00	1.00	0.00	.	.	.	1.00	.	.
2	0.52	0.00	0.19	0.04	25.02	5.7e-7	1.21	1.12	1.30
3	0.91	0.00	0.33	0.04	75.34	4.0e-18	1.39	1.29	1.50
4	1.00	0.00	0.36	0.04	89.40	3.2e-21	1.44	1.33	1.55

Figure 9-4
Model B: PAYMENT is treated as a FREE predictor with $1 used as the reference

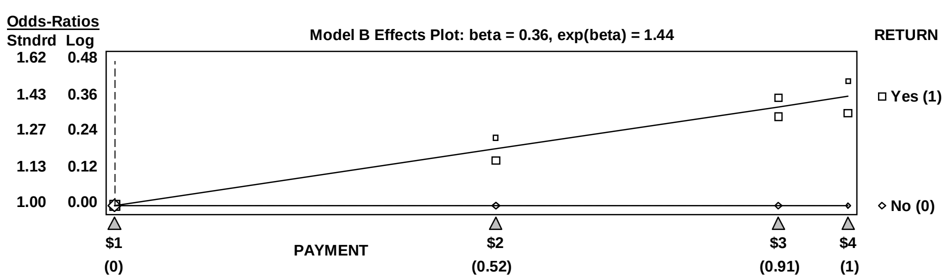


Figure 9-4 shows that persons receiving a $2 payment and for whom CALL = Yes(No) are estimated to be exp(0.52$\hat{\beta}_1$) = 1.21 times as likely to return the survey than persons receiving a $1 payment and for whom CALL = Yes(No). Similarly, given CALL = Yes(No), the effect of a $3 payment is estimated to be exp(0.91$\hat{\beta}_1$) = 1.39 and a $4 payment is exp($\hat{\beta}_1$) = 1.44 times as likely to return the survey than persons receiving a $1 payment (who were exposed to the same calling option).

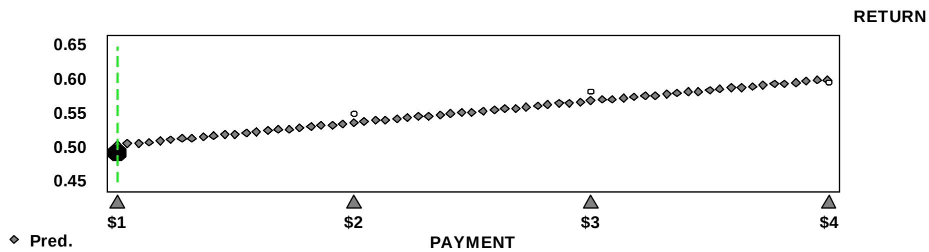
Since PAYMENT is treated as a **Free** scale (qualitative) variable, GOLDMineR estimates category-specific effects $\beta_k = \beta x^*_{1k}$, for each payment level k = 1, 2, 3,4 and lists them in a separate section of the output. By default, dummy contrast coding is used with the category allocated the 0 score (the *$1* PAYMENT category) being the one omitted from the regression (see "Category Weights" on page 119). The estimate for a *$2* payment, shown in Figure 9-3 as *Beta*(2) = 0.19, represents the increased likelihood of returning the survey associated with a *$2* payment vs. a *$1* payment. Examining the column titled *p-value* in Figure 9-3, we see that this effect is statistically significant ($p=5.7x10^{-7}$), and that $\hat{\beta}_3$ and $\hat{\beta}_4$ are also significantly different from zero.

To test the adequacy of the equidistant scoring assumption we compare the model fit statistics from the two models (see "A.10 Estimation Algorithm" on p. 228). Examining the output in Figure 9-1 and Figure 9-3 we see that Residual L^2 has been reduced from 9.45 with 5 d.f. under Model A to 2.38 with 3 d.f. under Model B, a reduction of 7.07 (with 5-3 = 2 d.f.) which is statistically significant at the p = .05 level. Hence, we reject the equidistant scaling assumption made by Model A in favor of Model B.

Suppose that we estimated only Model A. Would we still reject the equidistant scoring assumption? While Residual L^2 = 9.45 (with 5 d.f.) is an adequate fit (p = .09), a comparison of the observed and expected log-odds ratio in Figure 9-2 might lead us to reexamine the equidistant scoring assumption. Specifically, the observed log-odds ratios for persons who received a reminder call, as well as the ratios for those who did not receive a call, both appear <u>above</u> the corresponding expected ratios (i.e., above the effects line) for each of the two middle payment categories (*$2* and *$3* payments). Note the improvement in fit attained in Figure 9-4 when the PAYMENT scores are no longer restricted to be equidistant.

Figure 9-5 and Figure 9-6 illustrate partial Regression plots for PAYMENT given CALL under Model A. A careful examination of the pattern of residuals in these figures would also raise some questions about the correctness of the equidistant PAYMENT scores assumption. In particular, the two shaded circles show that the predicted return rates for the '*$1*, no call' (see Figure 9-5) and '*$4*, call' (see Figure 9-6) cells differ significantly from the observed rates. Figure 9-7 and Figure 9-8 show the closer fit to the observed return rates obtained under Model B where PAYMENT is treated as a qualitative variable (**Free** scale).

Figure 9-5
Model A: Predicted Probability of RETURN = 'Yes' by PAYMENT given CALL = 'No'



o = observed return rate (shaded means that observed rate is significantly different from expected)

Figure 9-6
Model A: Predicted Probability of RETURN = 'Yes' by PAYMENT given CALL = 'Yes'

Figure 9-7
Model B:Predicted Probability of RETURN = 'Yes' by PAYMENT given CALL = 'No'

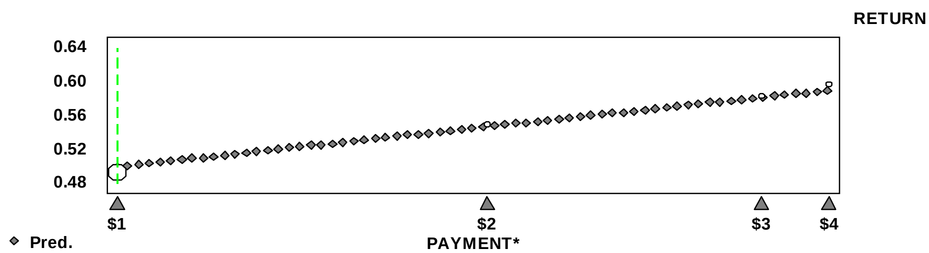
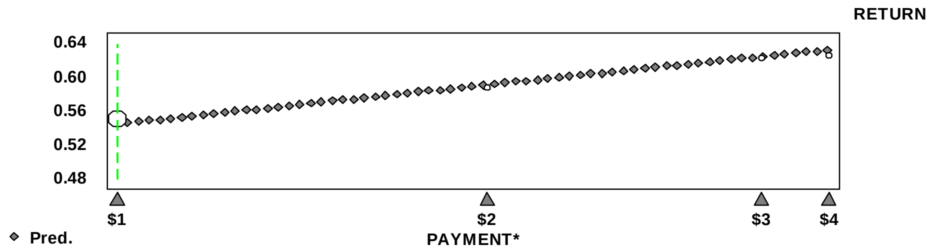


Figure 9-8
Model B: Predicted Probability of RETURN = 'Yes' by PAYMENT given CALL = 'Yes'



Should we accept Model B as our final model?

The category-specific parameter estimate section of Figure 9-3 confirmed that a *$2* payment (as well as *$3* and *$4* payments) are significantly better than a *$1* payment ($p = 5.7 \times 10^{-7}$). That is, all other factors being equal, persons receiving a *$2* payment are estimated to be 1.21 times as likely as those receiving a *$1* payment to return the survey (see the caption under Figure 9-4). However, it is not clear from Figure 9-3 whether the estimate of $\exp(\beta_4) = 1.44$ is significantly higher than the estimate of $\exp(\beta_3) = 1.39$. Similarly, from an examination of the Effects plot in Figure 9-4 we might ask the question of whether a *$4* payment is significantly better than a *$3* payment.

To test the effect of a *$4* payment relative to a *$3* payment we must change the reference category from the *$1* level to the *$3* level by the appropriate change in contrast coding. Once a model has been estimated, alternative contrast coding options may be selected to obtain different graphical views of the model and different category-specific parameter estimates, without reestimating the model (i.e., only the category-specific parameter estimate section of the output changes). Figure 9-9 and Figure 9-10 provide results for Model B when the reference is changed to the *$3* level. Note that the X-scores in Figure 9-9 are lower than those in Figure 9-3 by 0.91, the value of the original X-score shown in Figure 9-3 for the *$3* payment level).

Figure 9-9
Category-specific parameter estimates for PAYMENT under Model B where $3 is the reference

Category-Specific Parameter Estimates

PAYMENT (Free)	X-score	C-weights	Beta(k)	Std. Err.	Wald	p-value	exp(Beta)	Lower	Upper
$1	-0.91	0.00	-0.33	0.04	75.34	4.0e-18	0.72	0.67	0.78
$2	-0.39	0.00	-0.14	0.04	10.12	0.0015	0.87	0.80	0.95
$3	0.00	1.00	0.00	.	.	.	1.00	.	.
$4	0.09	0.00	0.03	0.04	0.59	0.44	1.03	0.95	1.13

Figure 9-10
*Model B: PAYMENT is treated as a **Free** predictor with $3 used as the reference to assess the significance of $4 vs. $3*

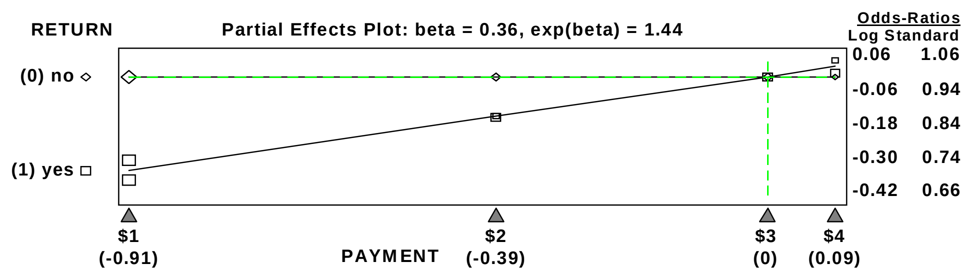


Figure 9-10 shows that persons receiving a $4 payment and for whom CALL = Yes(No) are estimated to be exp(0.09$\hat{\beta}_1$) = 1.04 times as likely to return the survey than persons receiving a $3 payment and for whom CALL = Yes(No).

Figure 9-9 shows that the effect of *$4* relative to *$3*, $\hat{\beta}_4$ = .034, is not significant at the .05 level (p = 0.44). This result suggests that we assign the same score to a *$3* and a *$4* payment and estimate a final model. One possibility for a final model is to recode the original datafile to combine payment levels *$3* and *$4* and estimate a model with PAYMENT treated as either Free or Fixed using equidistant scores. Recoding could also be implemented by using the Group feature in GOLDMineR which will automatically combine the *$3* and *$4* payment levels if 3 groups are specified (for more information on the Grouping feature, see “Variable Scaling Options” on page 90).

For the remainder of this section, we maintain separate categories for the *$3* and *$4* payment levels and assign **Fixed** PAYMENT scores of 0, 0.5, 1 and 1. That is, model C positions the *$2* payment equidistant between *$1* and *$3*, and positions the *$4* payment at the same place on the plot as the *$3* payment. Figure 9-11 summarizes the results and Figure 9-12 displays the effects plot under Model C.

Figure 9-11
*Results of model specifying **Fixed** PAYMENT scores: 0, 0.5, 1, 1*

y: RETURN (Fixed)	0	1			
Y-scores	0.00	1.00			
Alpha	0.00	0.01			
		$L^2(Y)$	df	p-value	Beta	exp(Beta)
PAYMENT (Fixed)		125	1	4.5e-29	0.35	1.41
CALL (Fixed)		42.6	1	6.6e-11	0.18	1.20
Association Summary		L^2	df	p-value	R^2	
Explained by Model		167	2	5.1e-37	0.00779	

Figure 9-12
*Effects Plot for Model C specifying **Fixed** PAYMENT scores: 0, 0.5, 1, 1*

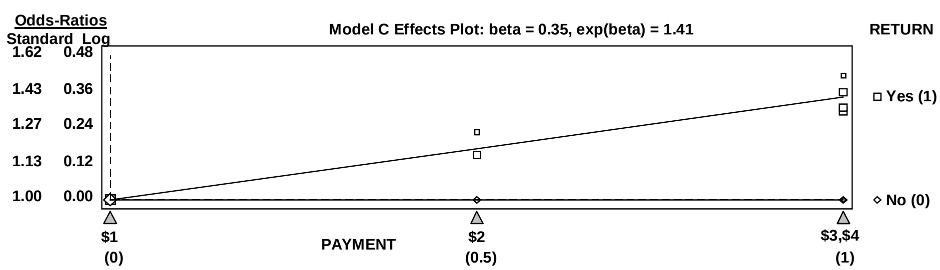


Figure 9-12 shows that persons receiving a $3 or $4 payment and for whom CALL = Yes(No) are estimated to be exp($\hat{\beta}_1$) = 1.41 times as likely to return the survey than persons receiving a $1 payment and for whom CALL = Yes(No).

Under Model C, each additional *$1* PAYMENT up to *$3* is expected to increase the likelihood of RETURN. A payment of *$4* is not expected to lift response above that of a *$3* payment.