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Markov Tutorial #1: Latent GOLD Longitudinal Analysis of Brand Loyalty

Overview

Goals – To show how Latent GOLD's new latent Markov module can be useful in measuring brand loyalty and more generally in handling autocorrelation in longitudinal data.

- Distinguish between 'wide' and 'long' formatted data
- Use Latent GOLD 5.0 to introduce Latent Class (LC) Growth and Latent Markov (LM) Models
- Describe the use of Model Fit Statistics including new Longitudinal Bivariate Residuals (LBVRs)
- Show how to generate brand purchase forecasts for future time points

The Data

The data consists of a dichotomous indicator measured for each of 631 respondents at each of 5 equidistant time points: (Y) Choose Brand A? 1=Yes or 0=No. The variable 'id' is used to distinguish each of the 2^5 = 32 response patterns, and the variable 'freq' is used as a case weight[1]. For example, 352 respondents purchased brand A at all 5 time points, Y=(1,1,1,1,1). In total we have N=631 respondents.

A

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1 : y.1 | 1 | Visible: 7 of 7 Variables

	id	freq	y.1	y.2	y.3	y.4	y.5
1	1	352	1	1	1	1	1
2	2	44	0	1	1	1	1
3	3	26	1	0	1	1	1
4	4	14	0	0	1	1	1
5	5	20	1	1	0	1	1
6	6	5	0	1	0	1	1
7	7	7	1	0	0	1	1
8	8	12	0	0	0	1	1
9	9	14	1	1	1	0	1
10	10	5	0	1	1	0	1
11	11	5	1	0	1	0	1
12	12	5	0	0	1	0	1

Data View | Variable View

IBM SPSS Statistics Processor is ready | Weight On

B

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Visible: 4 of 4 Variables

	id	y	freq	time
1	1	1	352	1
2	1	1	352	2
3	1	1	352	3
4	1	1	352	4
5	1	1	352	5
6	2	0	44	1
7	2	1	44	2
8	2	1	44	3
9	2	1	44	4
10	2	1	44	5

Data View | Variable View

IBM SPSS Statistics Processor is ready | Weight On

Figure 1. (A) Wide format (response patterns id=1,2,…,12 shown), and (B) Long format (response patterns id=1,2 shown).

[1] Rather than using a case weight, individual case level data (with 631 * 5 = 3155 records) could be analyzed by Latent GOLD.

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We begin by developing simple LC growth models for these data using the wide format. We then re-estimate these same models using the more flexible long format. The long format can also be used to estimate latent Markov models[2] which fit better by including transition probability parameters.

Latent Class (LC) Growth Models

The overly simplistic null (1-class) latent growth model assumes mutual independence between brand choice behavior at the various times. This model assumes that a person's purchase behavior at time t is independent of their previous purchase behavior (i.e., no autocorrelation) and that everyone has the same probability P_t of buying brand A at time t. Under this model, the maximum likelihood estimate for P_t is the overall sample proportion who bought brand A at time t, t=1,2,3,4,5. These proportions are plotted in Figure 2A, which show an increase in market share.

The 2-class latent growth model is a simple extension of the null (1-class) model that accounts for autocorrelation by postulating 2 latent classes, respondents in each class having different purchase probabilities. Mutual independence is assumed for each of the 2 classes. Maximum likelihood estimates for the purchase probabilities for each class as well as each person's class assignment can be obtained with Latent GOLD.

Figure 2B shows the resulting purchase probabilities for each class. Persons classified into Class 1 might be named 'loyal to brand A', as the plot shows that this class has probability of about .9 of buying brand A at each time point, while class 2 has much lower probabilities. From the Profile output (Figure 3), we see that this 'loyal to brand A' cluster consists of 78.7% of the respondents.

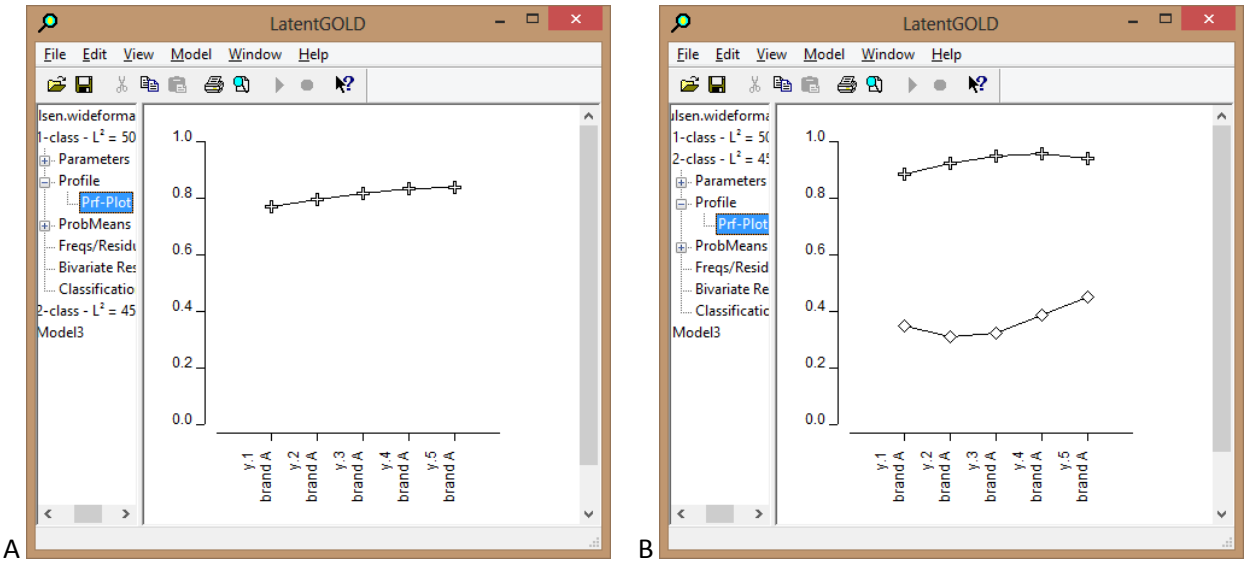

Figure 2. Predicted share trends: probability of buying brand A as predicted under (A) 1-class (null) model and (B) 2-class Latent Growth Model over 5 equidistant points in time.

[2] These data were analyzed earlier using Markov models by Poulsen (1990).

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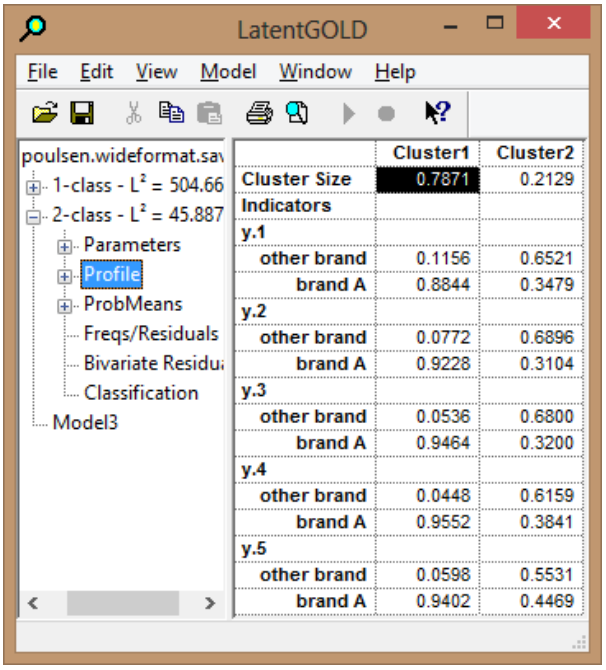

	Cluster1	Cluster2
Cluster Size	0.7871	0.2129
Indicators		
y.1		
other brand	0.1156	0.6521
brand A	0.8844	0.3479
y.2		
other brand	0.0772	0.6896
brand A	0.9228	0.3104
y.3		
other brand	0.0536	0.6800
brand A	0.9464	0.3200
y.4		
other brand	0.0448	0.6159
brand A	0.9552	0.3841
y.5		
other brand	0.0598	0.5531
brand A	0.9402	0.4469

Figure 3. Profile output for 2-cluster model.

Figures 2A, 2B, and 3 were obtained from a Latent GOLD cluster analyses of these data formatted using the *wide* format where there is one record per response pattern and each time point is a separate variable (column). Details of this analysis are provided in the Appendix. For an introduction to latent class modeling of data in the wide format see Latent GOLD Tutorial 1.

Estimating LC Growth Models with Long Format Data

The long file format allows more flexibility in analyzing longitudinal data and allows us to estimate a variety of latent Markov models and to obtain longitudinal bivariate residuals (BVRs), new in Latent GOLD version 5.0:

- Open file 'poulsen.longformat.sav', and set up a regression model as shown in Figure 5A.
- From the Output tab request additional output 'Classification – Posterior', and 'Frequency/Residuals' (Figure 4B).
- Click Estimate

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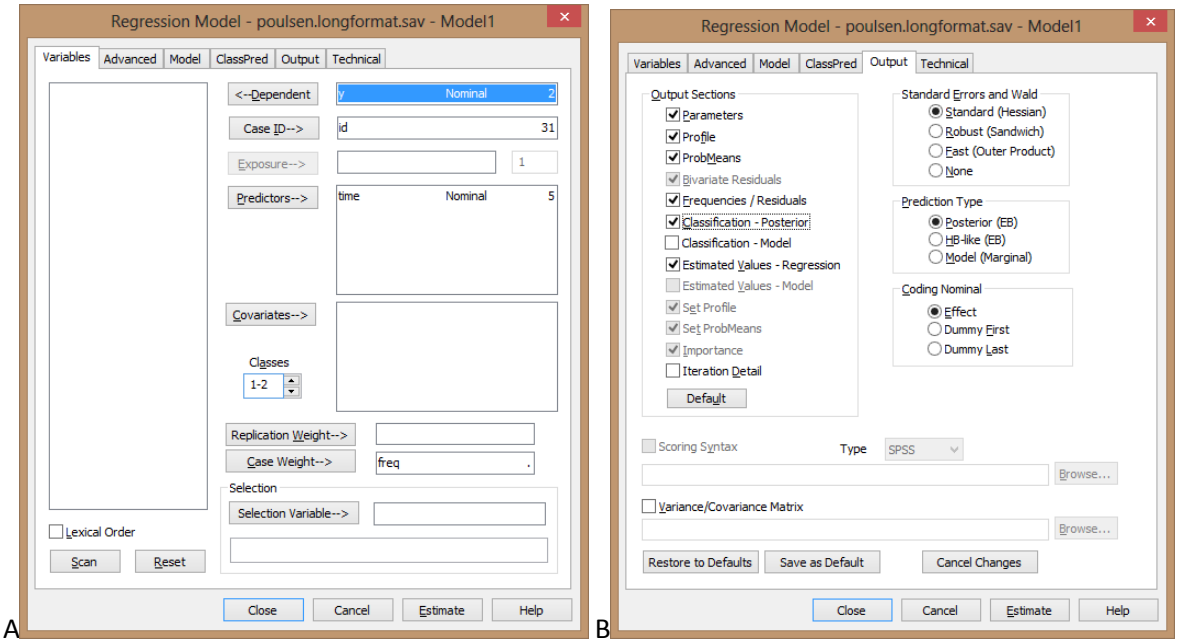

Figure 4. Regression model setup using long format data, (A) Variables tab, and (B) Output tab.

After the estimation has been completed,

- Rename 'Model1' to '1-class Regression' and 'Model2' to '2-class Regression'.
- Click on the datafile name to display the Model Summary output
- Right click and use the Model Summary Display to remove LL, and BIC(LL) and add BIC(L^2).

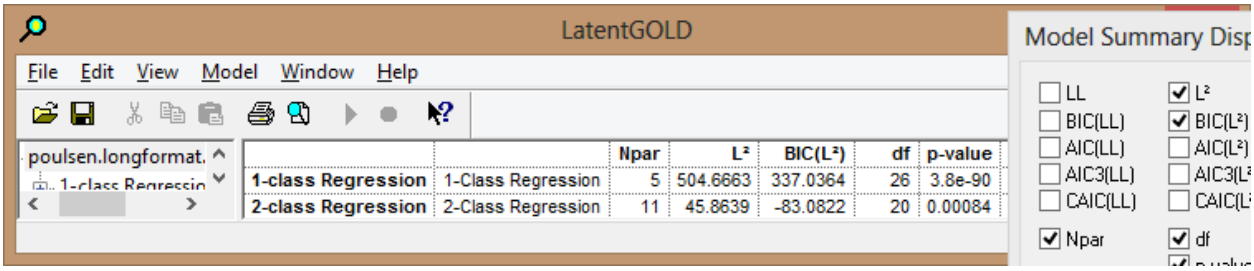

Figure 5. Model Summary output

Note that the 1-class model provides a very poor fit (p = 3.8E-90) and that the 2-class latent growth model provides substantial improvement (L^2 is reduced from 504.7 to 45.9). From the numbers in the column 'Npar' (Figure 5) we can calculate that there are 11 – 5 = 6 additional parameters in the 2-class model (1 parameter for the size of class 2 plus the 5 time-specific purchase probabilities associated with this second class). Despite this improvement, the model fit p-value = .00084, which means that this model still fails to explain all the autocorrelation in the data.

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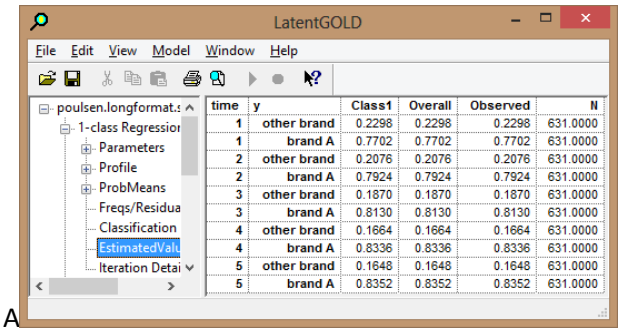
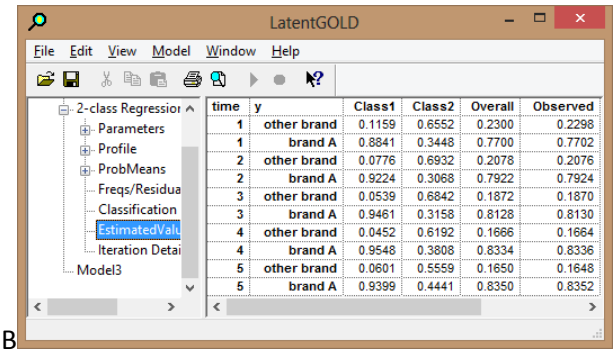
Estimates based on these models for the purchase probabilities were plotted in Figure 2A and Figure 2B. Since the current analyses are based on the long format data, purchase probability estimates can be found in the Estimated Values output. Figure 6 shows the Estimated Values output for both models.

A



time	y	Class1	Overall	Observed	N
1	other brand	0.2298	0.2298	0.2298	631.0000
1	brand A	0.7702	0.7702	0.7702	631.0000
2	other brand	0.2076	0.2076	0.2076	631.0000
2	brand A	0.7924	0.7924	0.7924	631.0000
3	other brand	0.1870	0.1870	0.1870	631.0000
3	brand A	0.8130	0.8130	0.8130	631.0000
4	other brand	0.1664	0.1664	0.1664	631.0000
4	brand A	0.8336	0.8336	0.8336	631.0000
5	other brand	0.1648	0.1648	0.1648	631.0000
5	brand A	0.8352	0.8352	0.8352	631.0000

B



time	y	Class1	Class2	Overall	Observed
1	other brand	0.1159	0.6552	0.2300	0.2298
1	brand A	0.8841	0.3448	0.7700	0.7702
2	other brand	0.0776	0.6932	0.2078	0.2076
2	brand A	0.9224	0.3068	0.7922	0.7924
3	other brand	0.0539	0.6842	0.1872	0.1870
3	brand A	0.9461	0.3158	0.8128	0.8130
4	other brand	0.0452	0.6192	0.1666	0.1664
4	brand A	0.9548	0.3808	0.8334	0.8336
5	other brand	0.0601	0.5559	0.1650	0.1648
5	brand A	0.9399	0.4441	0.8350	0.8352

Figure 6. Estimated values for (A) 1-class regression, and (B) 2-class regression.

For the 2-class model (Figure 6B) the output corresponds[3] to the Profile output from the corresponding analysis based on wide format data (compare Figure 6B with Figure 3), which were plotted in Figure 2B. As mentioned above, class 1 represents the 'loyal to brand A' class.

To estimate a restricted 2-class regression model such that the purchase probability for each class is constant over time,

- Double click on '2-class Regression' to open the regression model dialog box
- Click on the Model tab
- Left click to highlight the time effect cells for classes 1 and 2 as shown in Figure 7
- Right click and select 'No Effect' from the pop up menu (see Figure 7)
- Click Estimate

[3] As explained in the appendix, small differences are due to slightly different ways that the Bayes constant is implemented in the cluster and regression models.

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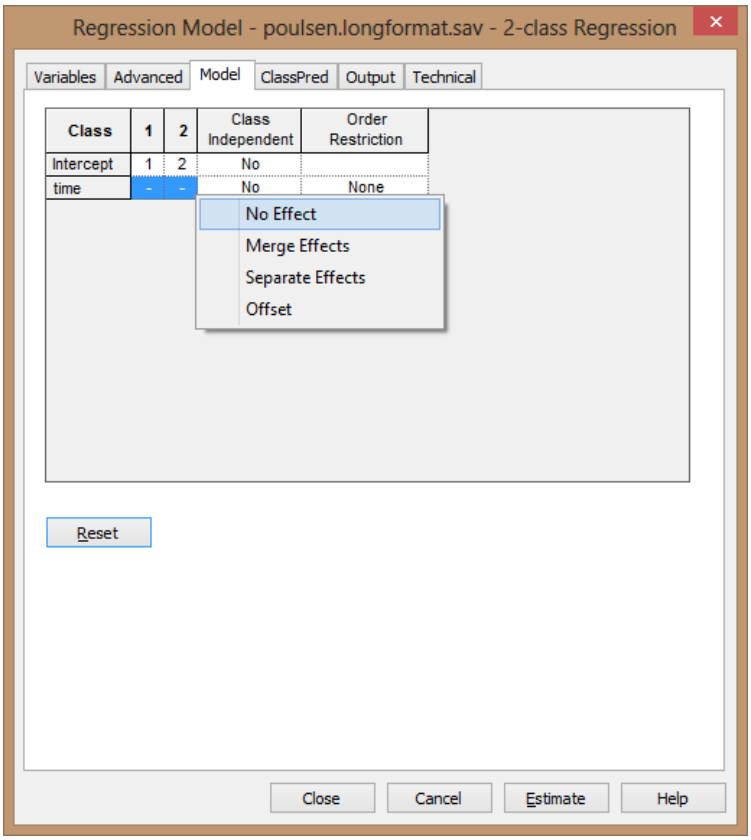

Figure 7. Using the Model tab to restrict the purchase probability for each class to be constant over time.

- Click on datafile name to display the Model Summary output

The lower BIC for the restricted 2-class model supports the time constant purchase probabilities, although this model does not alter the fact that this 2-class regression model fails to provide an adequate fit to the data (p=1.1E-5).

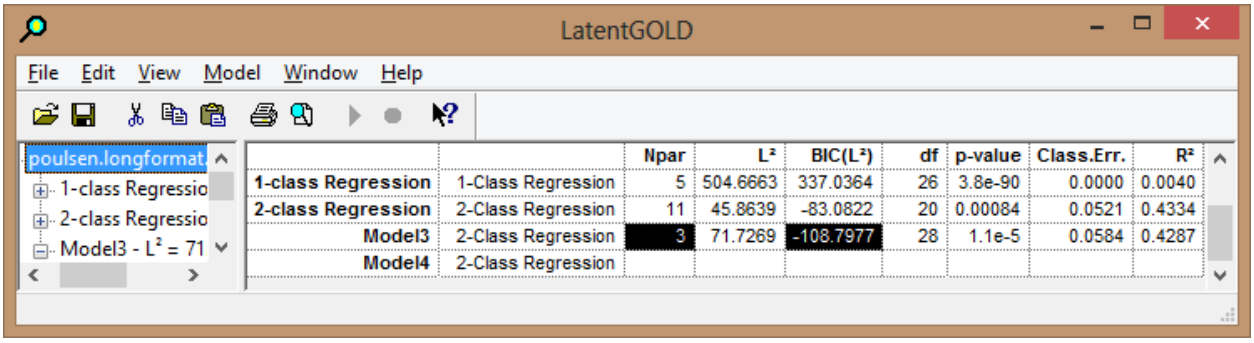

Figure 8. Model Summary output comparing the restricted 2-class model (Model 3) with the other 2 LC growth models.

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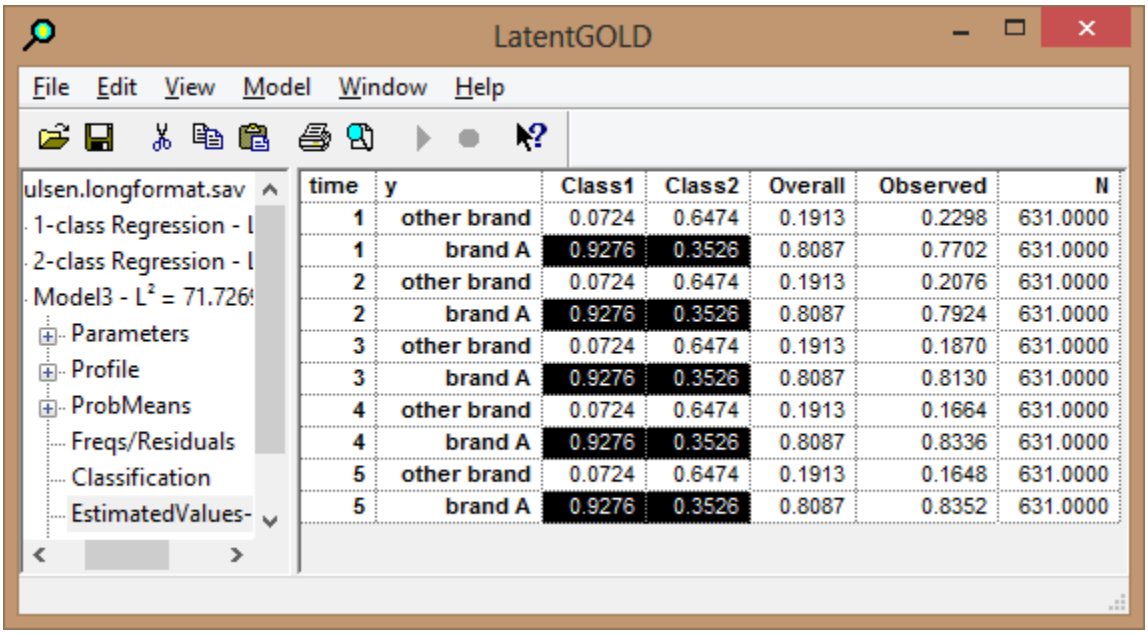
To see these purchase probabilities,

- Click on EstimatedValues to display the Estimated Values output



time	y	Class1	Class2	Overall	Observed	N
1	other brand	0.0724	0.6474	0.1913	0.2298	631.0000
1	brand A	0.9276	0.3526	0.8087	0.7702	631.0000
2	other brand	0.0724	0.6474	0.1913	0.2076	631.0000
2	brand A	0.9276	0.3526	0.8087	0.7924	631.0000
3	other brand	0.0724	0.6474	0.1913	0.1870	631.0000
3	brand A	0.9276	0.3526	0.8087	0.8130	631.0000
4	other brand	0.0724	0.6474	0.1913	0.1664	631.0000
4	brand A	0.9276	0.3526	0.8087	0.8336	631.0000
5	other brand	0.0724	0.6474	0.1913	0.1648	631.0000
5	brand A	0.9276	0.3526	0.8087	0.8352	631.0000

Figure 9. Estimated values for restricted 2-class regression show purchase probabilities for each class.

We see that the probability of purchasing brand A is .9276 for the 'loyal to brand A' class (Class1), and .3526 for the other class (Class2). To see which response patterns are classified as being in the 'loyal to brand A' class,

- Click on Classification

From the classification output (Figure 10) we see that ID=1-3, 5, and 9 are among those response patterns most consistent with being loyal to brand A ('Modal'=1), and patterns ID=4, 6-8, and 10-11 are more likely to be associated with the other class ('Modal'=2).

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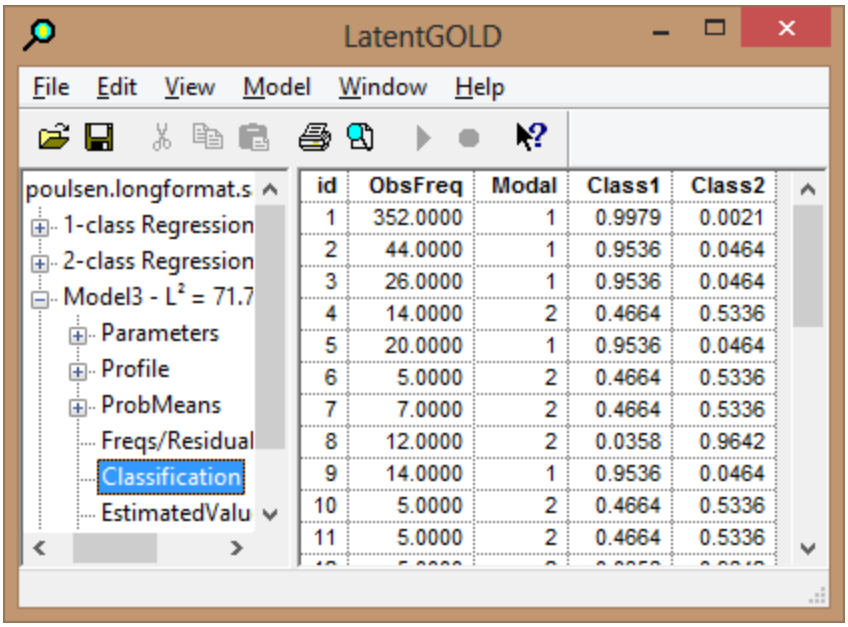

Figure 10. Classification output for the restricted 2-class growth model.

Estimating Latent Markov Models

As shown in Figure 29 of the Appendix, the reason that the 2-class model fails to provide an adequate fit is that the 2 classes fail to fully explain the first order autocorrelation in these data.

Latent Markov models differ from traditional latent class models in that they contain *transition probability parameters,* which accounts for first order autocorrelations directly by allowing the latent category that a person is in to change over time. Such *dynamic* latent categories are called *states*. A 2-state latent Markov (LM) model differs from a 2-class latent growth model in 2 primary ways:

- While persons in the loyal class of the 2-class latent growth model remain in the *loyal* class throughout the 5 time points, persons classified by the latent Markov model, at say time 1, into the *loyal state*, may change to the *non-loyal* state at some later time.
- Unlike LC growth models, the LM model explicitly includes the probability of changing from 1 state at time t-1 to some other state at time t (*transition probabilities*) as model parameters.

Because these transition probability parameters account for first order autocorrelation directly, we will see that the LM model provides a better fit to these data.

To setup the (time homogeneous) latent Markov model with 2 states:

- Right click on Model4 and select 'Markov'
- Setup the Variables tab as shown below (Figure 11A)
- In the Advanced tab, change the number of classes to '1'

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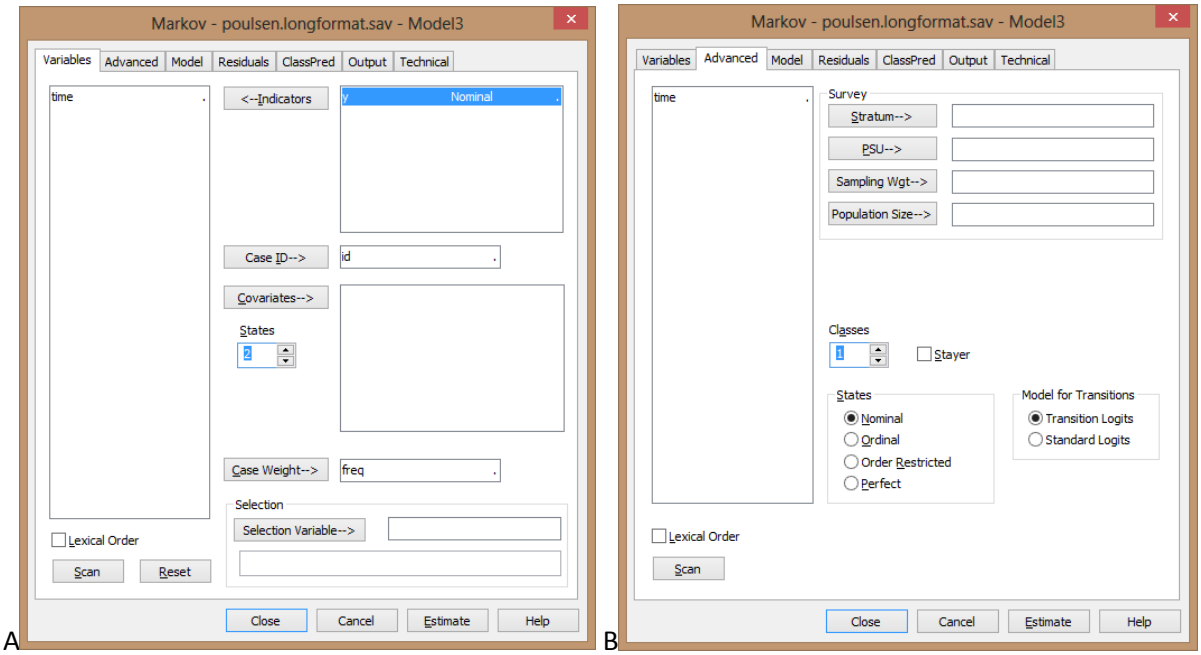
- Click Estimate



A B

Figure 11. (A) Latent Markov Variables tab, and (B) Advanced tab.

- Click Estimate
- Rename 'Model4' to 'latent Markov'

As expected the LM model provides a much better fit to these data, p=.77. Also, the LM model is very parsimonious, containing only 5 parameters, the same number of parameters as the 1-class model!

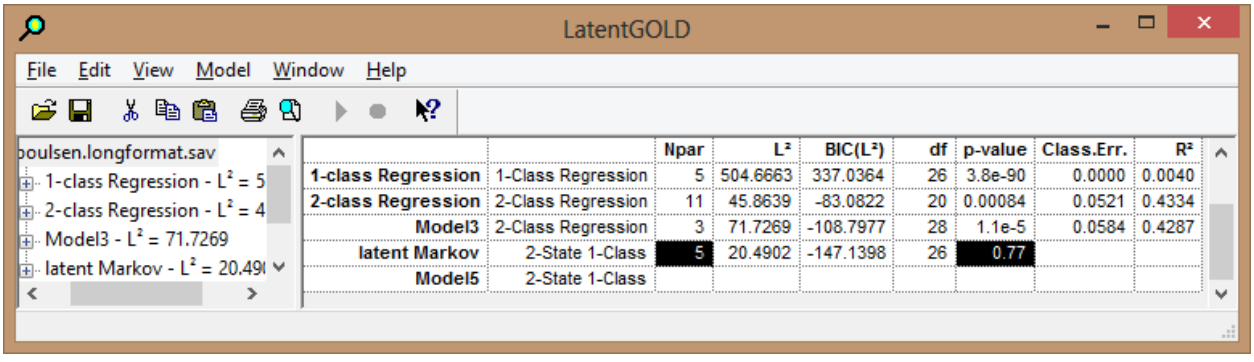


Figure 12. Model Summary output showing that the latent Markov model fits these data (p=.77).

As expected given the good fit (p-value =.77), the frequency output shows that all standardized residuals have magnitude less than 2.

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The 5 LM estimated probability parameters can be seen in the EstimatedValues-Model output:

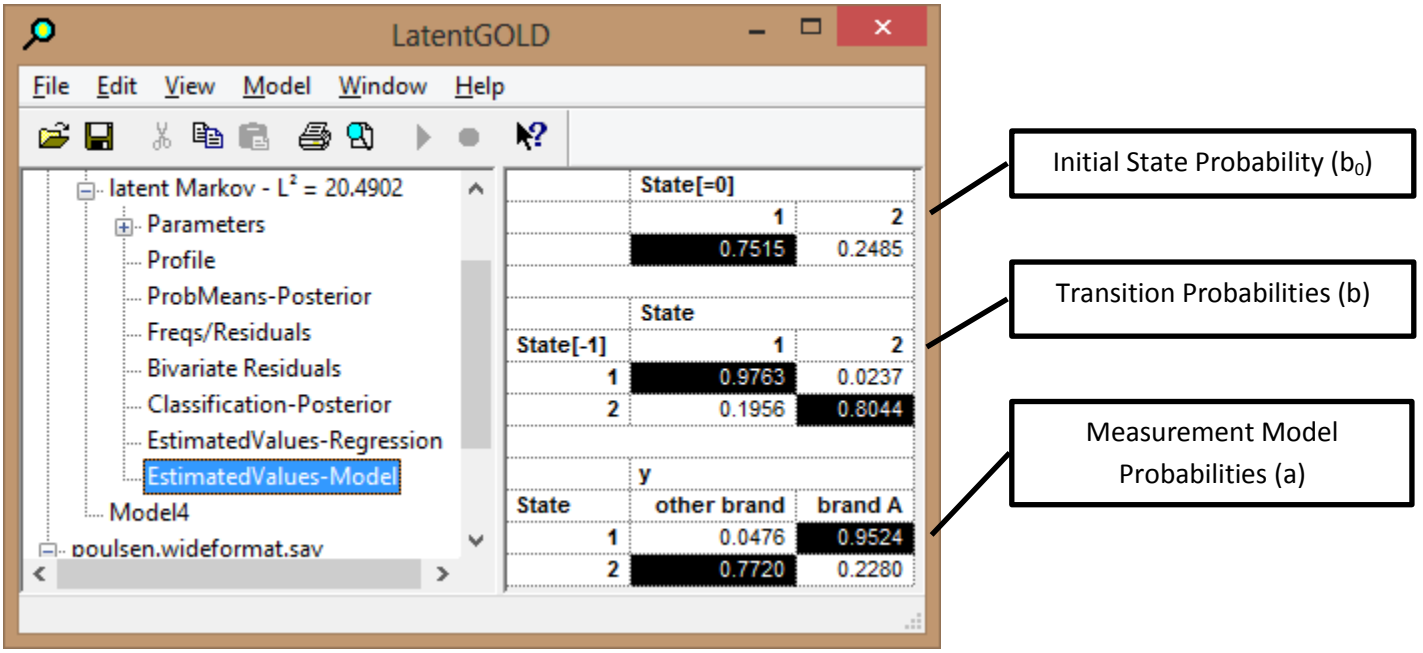

Figure 13. EstimatedValues-Model output.

The 5 parameters, identified in Figure 14 as parameters b0, a=(a[1], a[2]), and b=(b[1], b[2]), consist of the following:

1 distinct *initial state probability* parameter: The probability of being in the loyal state initially is b_0 =.7515. Since the State 2 probability can be computed from the state 1 probability (= 1 – b_0), we count only 1 (distinct) initial state probability parameter.

2 distinct *measurement model probabilities*: Persons in State 1 purchase brand A a[1] = 95.24% of the time. Thus, State 1 represents the 'loyal to brand A' state. Persons in State 2 purchase brand A a[2] = 22.8% of the time (thus, purchase the other brand 1- a[2] = 77.2% of the time).

2 distinct *transition probabilities*: Given a case is in 'loyal to brand A' state (State 1) at time t-1, the probability of remaining in this state at time t is b[1] = .9763. Given a case is in the other state (State 2) at time t-1, the probability of remaining in this state at time t is b[2] = .8044, and thus b[2] = 1 - .8044 = .1956 change to the loyal-to-brand-A state. As we will see later (Figure 21), the higher loyalty retention rate implies an increasing share of loyal customers over time.

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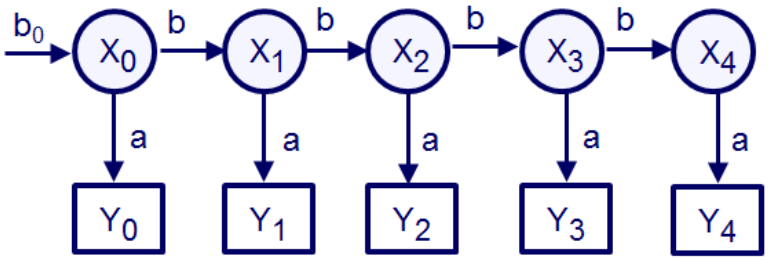

Figure 14. Path diagram for latent Markov Model where X denotes the latent states.

The only difference between this 2-state latent Markov model and the restricted 2-class regression model (Model3) is the inclusion of the 2 transition probability parameters. To see this, we will now estimate a restricted latent Markov model by restricting the 2 transition probability parameters to equal 1 (i.e., restrict the transition probability matrix to the identity matrix) and show that the resulting model is equivalent to the restricted 2-class regression model. That is, we restrict the states to be classes, meaning that respondents are not allowed to change from 1 state to another.

- Double click on 'latent Markov' to open the latent Markov dialog box
- Click on the Advanced tab
- Check the 'Stayer' box to impose the no-change-in-state restrictions.
- Click Estimate

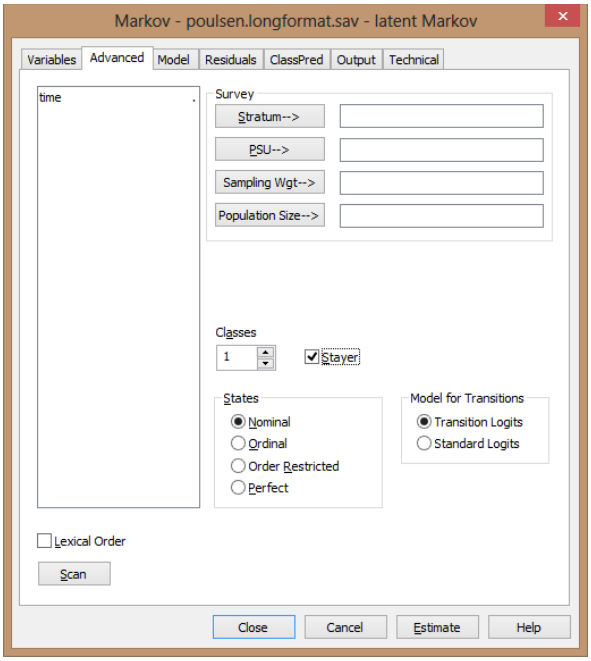

Figure 15. Advanced tab showing how to implement the 'Stayer' (no-change-in-state) restriction.

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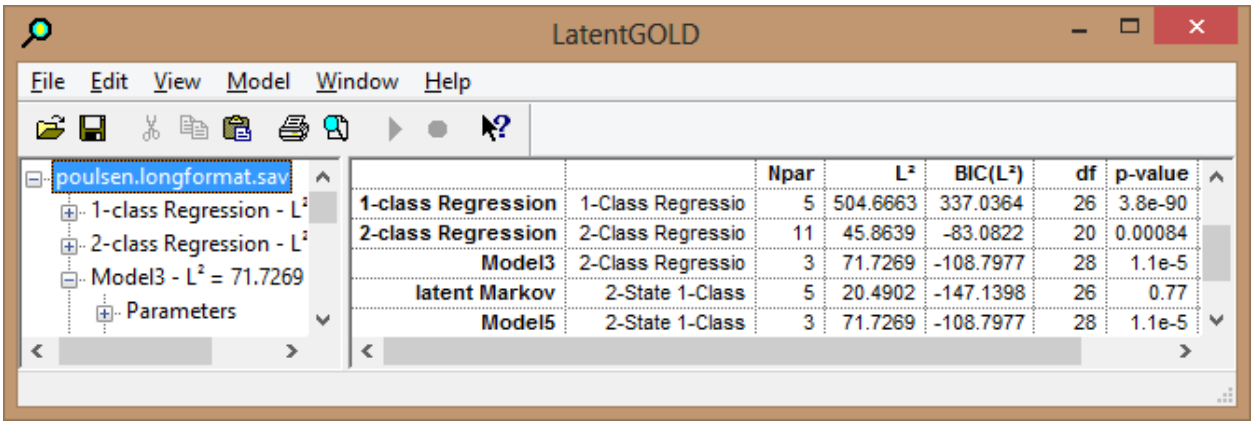
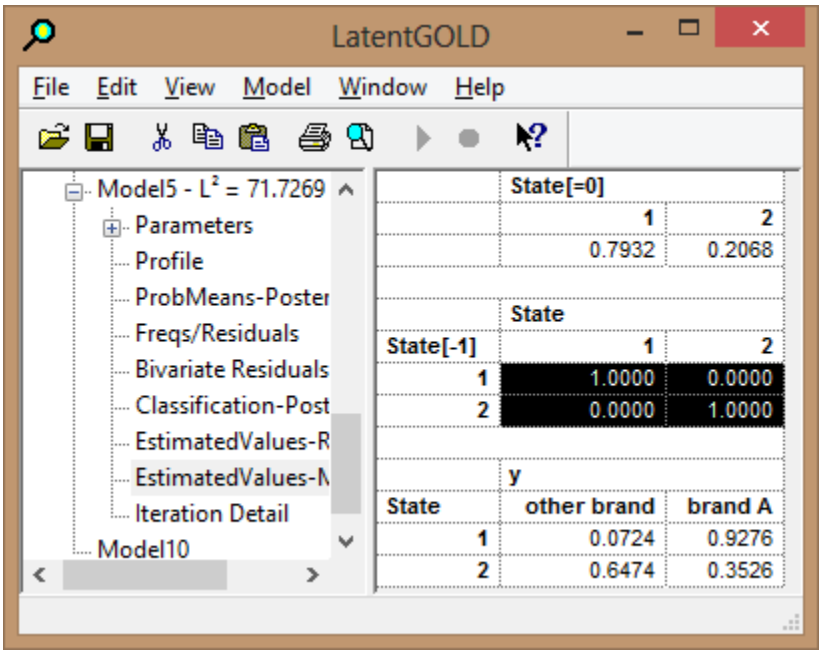


		Npar	L²	BIC(L²)	df	p-value
1-class Regression	1-Class Regressio	5	504.6663	337.0364	26	3.8e-90
2-class Regression	2-Class Regressio	11	45.8639	-83.0822	20	0.00084
Model3	2-Class Regressio	3	71.7269	-108.7977	28	1.1e-5
latent Markov	2-State 1-Class	5	20.4902	-147.1398	26	0.77
Model5	2-State 1-Class	3	71.7269	-108.7977	28	1.1e-5

Figure 16. Summary Output showing the equivalence between Model3 and Model5.

To verify that the transition probabilities implement the 'Stayer' (no-change-in-state) restriction,

- Click on 'EstimatedValues-Model' for Model5



	State[=0]	
	1	2
	0.7932	0.2068

	State	
State[-1]	1	2
1	1.0000	0.0000
2	0.0000	1.0000

	y	
State	other brand	brand A
1	0.0724	0.9276
2	0.6474	0.3526

Figure 17. EstimatedValues-Model output showing the transition probabilities in the stayer-restricted latent Markov model correspond to the identity matrix.

Longitudinal bivariate residuals (LBVRs) for the restricted and unrestricted 2-state latent Markov models show that inclusion of the transition probability parameters allow the first order autocorrelation to be explained much better by the (unrestricted) 'latent Markov' model than the corresponding latent class growth model (Model5). Lag1 LBVRs = .0115 for the latent Markov model vs. 16.186 for the LC growth model (see Figure 18). As a result, the overall model fit is now adequate (p=.77).

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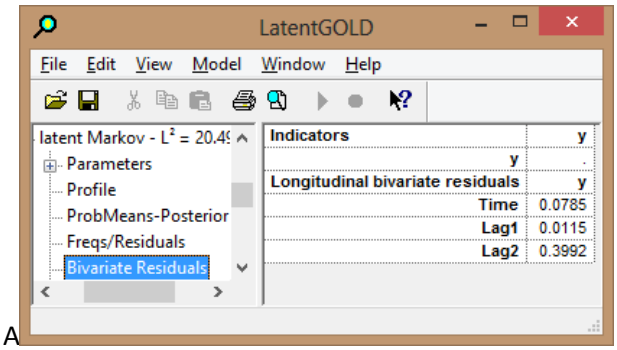
A 

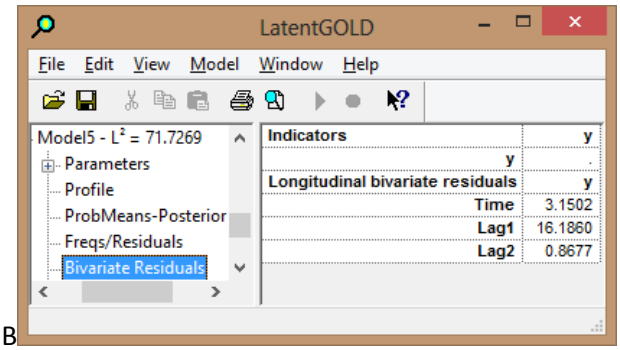
B 

Figure 18. Comparison of longitudinal bivariate residuals for (A) latent Markov model, and (B) latent class growth model.

Obtaining Forecasts from Latent Markov Models

In addition to accounting for first order autocorrelation directly, the transition probability parameters latent Markov models can be used to predict future time points. For example, to obtain forecasts for 5 future time points (time=6-10) we add the 5 additional time points to the input data file, which have missing values for Y as shown in Figure 19.

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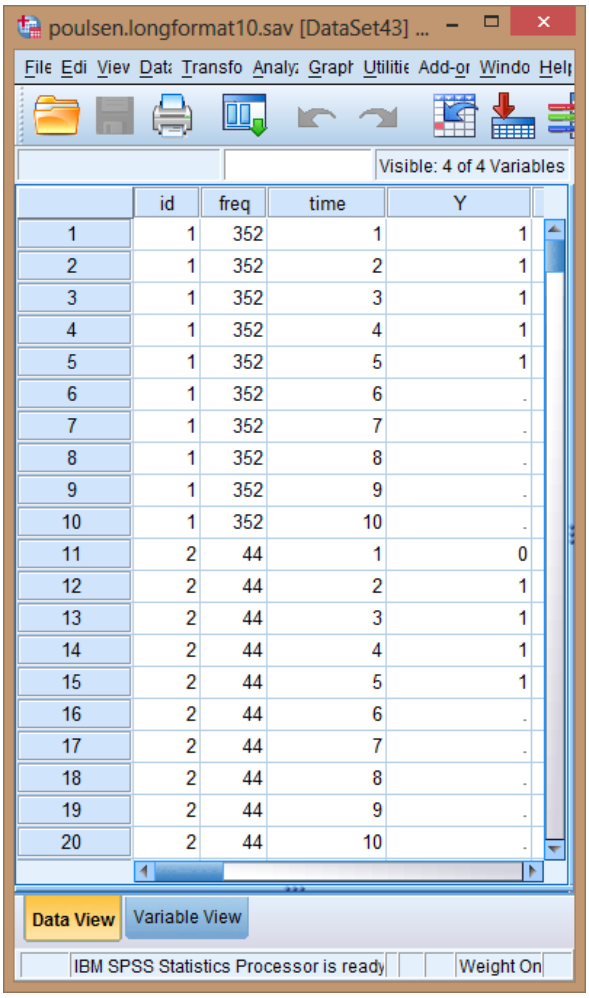

	id	freq	time	Y
1	1	352	1	1
2	1	352	2	1
3	1	352	3	1
4	1	352	4	1
5	1	352	5	1
6	1	352	6	.
7	1	352	7	.
8	1	352	8	.
9	1	352	9	.
10	1	352	10	.
11	2	44	1	0
12	2	44	2	1
13	2	44	3	1
14	2	44	4	1
15	2	44	5	1
16	2	44	6	.
17	2	44	7	.
18	2	44	8	.
19	2	44	9	.
20	2	44	10	.

Figure 19. Updated data containing additional records for future time points 6-10.

To obtain additional useful 'ProbMeans' output, we will include 'time' as an inactive covariate.

- Open file 'poulsen.longformat10.sav', right click on Model1 and select 'Markov'
- Setup the latent Markov model once again as shown below (Figure 20A).
- In ClassPred tab request 'Classification – Posterior', and 'Predicted Values' to be output to a file, and specify the outfile name 'Forecast.sav' (Figure 20B)
- Click Estimate

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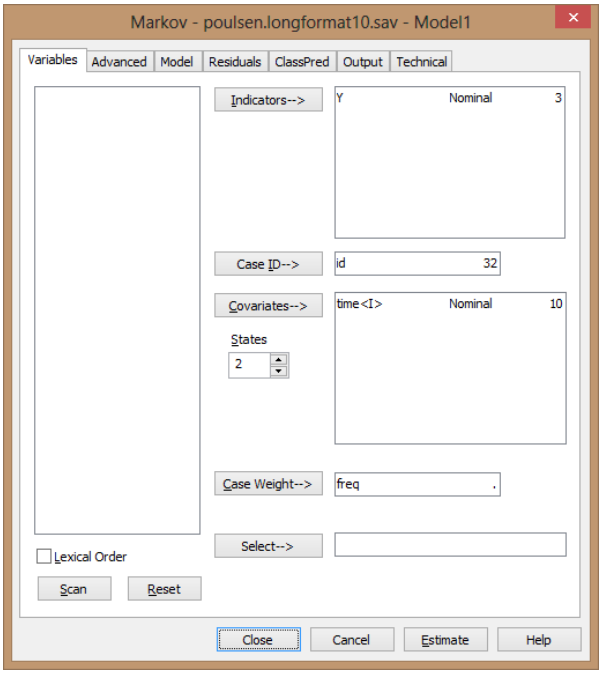
A 

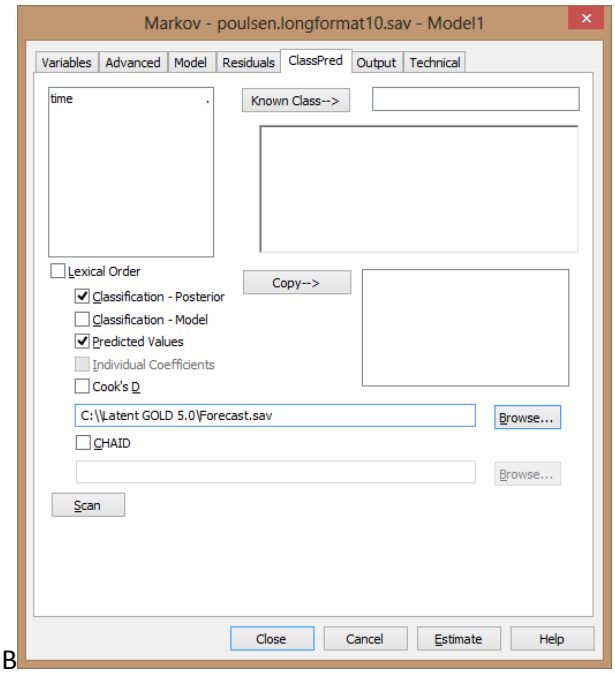
B 

Figure 20. Latent Markov model setup for updated data, (A) Variables tab, and (B) ClassPred tab.

To view the predicted trend in the loyal-to-brand-A state over time,

- Click 'ProbMeans-Posterior'

Note that the percentage of persons who are in the loyal-to-brand-A state increases from 75.19% in time 1 to 78.56% in time 2, to 83.89% in time 5 and the forecast under this model is for further increase up to 87.65% at time 10 (see Figure 21).

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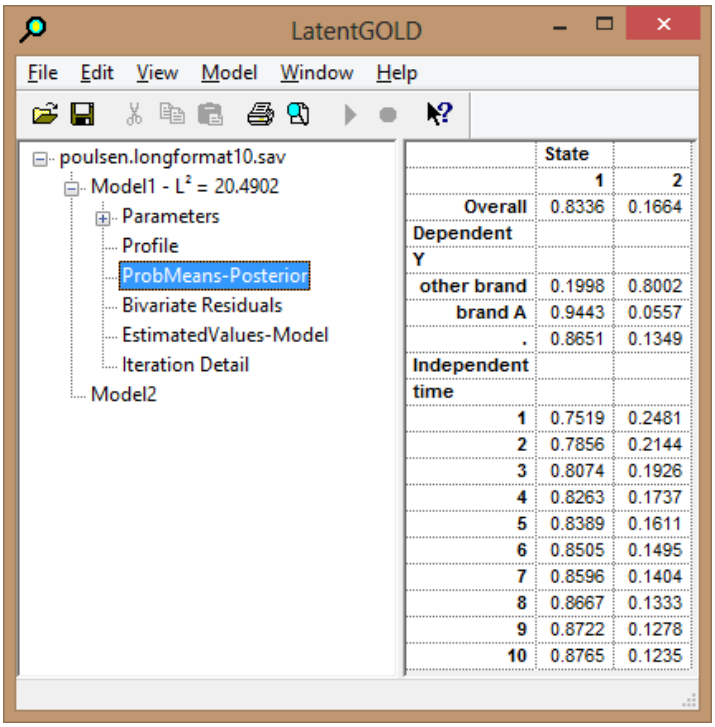

	State	
	1	2
Overall	0.8336	0.1664
Dependent		
Y		
other brand	0.1998	0.8002
brand A	0.9443	0.0557
.	0.8651	0.1349
Independent		
time		
1	0.7519	0.2481
2	0.7856	0.2144
3	0.8074	0.1926
4	0.8263	0.1737
5	0.8389	0.1611
6	0.8505	0.1495
7	0.8596	0.1404
8	0.8667	0.1333
9	0.8722	0.1278
10	0.8765	0.1235

Figure 21. Output showing state probabilities by time including future time points 6-10.

To plot this trend:

- Click 'Profile-Longitudinal'
- Click the '+' to expand the output listing
- Click 'Longitudinal-Plot
- Right click anywhere on the plot to open the Plot Control
- Check the box 'Overall' (see Figure 22)

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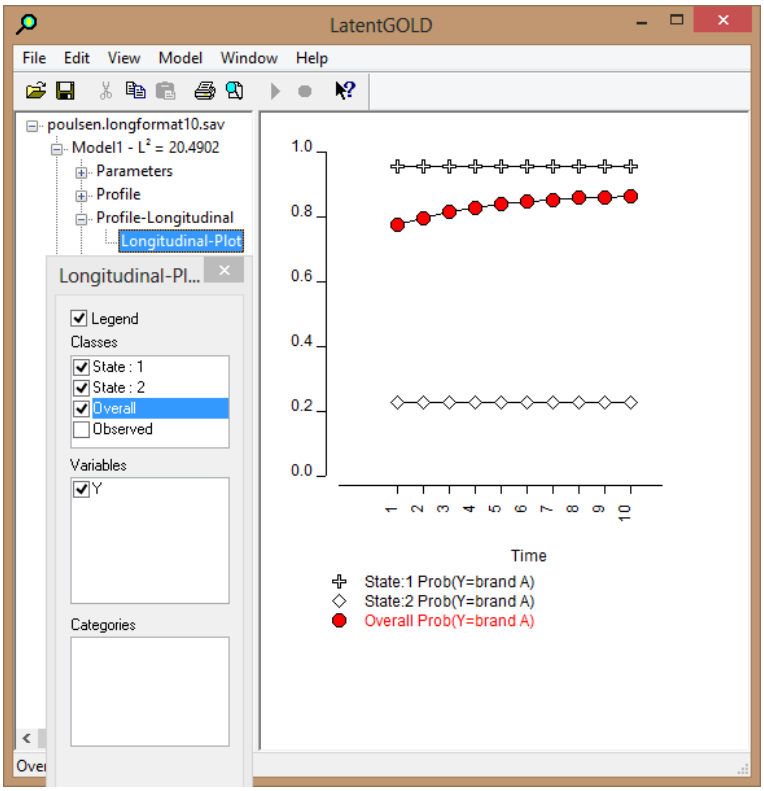

Figure 22. Longitudinal-Profile plot

The output file contains various predictions including the predicted probability of being in the loyal state at each point in time for each response pattern.

In longitudinal data analysis Tutorial 2, we provide an example of data that requires a more complex mixture latent Markov model to provide an adequate fit. The more complex model contains both time heterogeneous transitions and transitions that differ for two latent classes.

Additional Resources

For more details on using LC and LM models to analyze longitudinal data, see Vermunt, Tran, Magidson (2008). "Using latent class models in longitudinal research", chapter 23 in S. Menard (Ed), *Handbook of Longitudinal Research: Design, Measurement, and Analysis*, 373-385. Academic Press: Burlington, MA. http://www.statisticalinnovations.com/technicalsupport/VermuntTranMagidson2006.pdf

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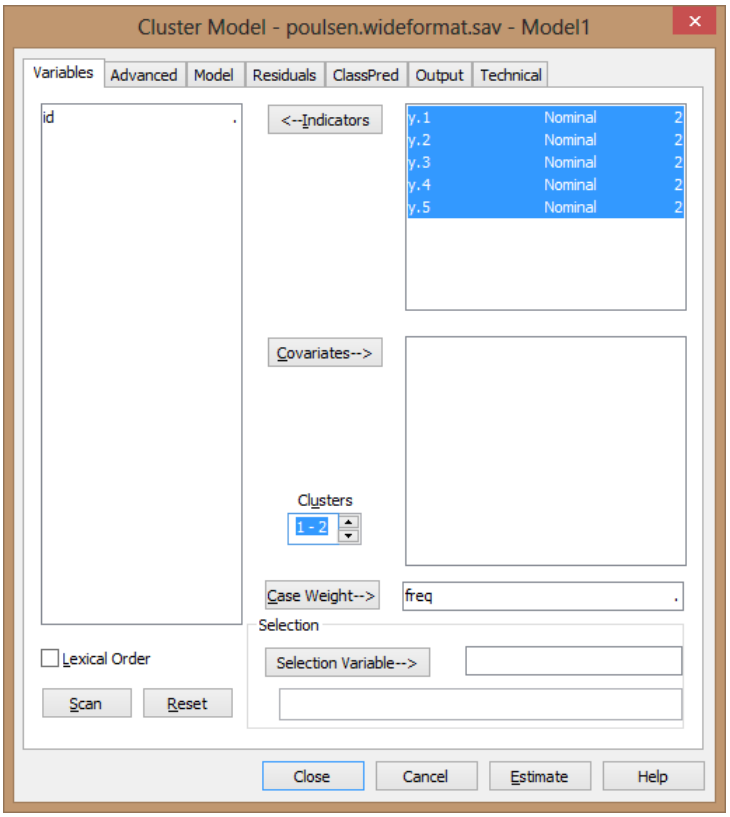
Poulsen, C. S. 1990. Mixed Markov and latent Markov modeling applied to brand choice behavior. International Journal of Research in Marketing, 7(1) 5–19.

Appendix

Estimating Latent Class (LC) Growth Models Using Wide Formatted Data

We will use Latent GOLD's Cluster module to estimate 1 and 2-class models, and interpret these as latent class (LC) growth models – providing different growth patterns (for each latent class) for the probability of purchasing Brand A.

- Open file 'poulsen.wideformat.sav', and use the Cluster module, setup[4] as shown in Figure 23
- In the Output tab request the additional output 'Classification – Posterior', and 'Frequency/Residuals' output (Figure 24)
- Click Estimate



[4] For further assistance see latent class tutorial #1

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Figure 23. Cluster model setup.

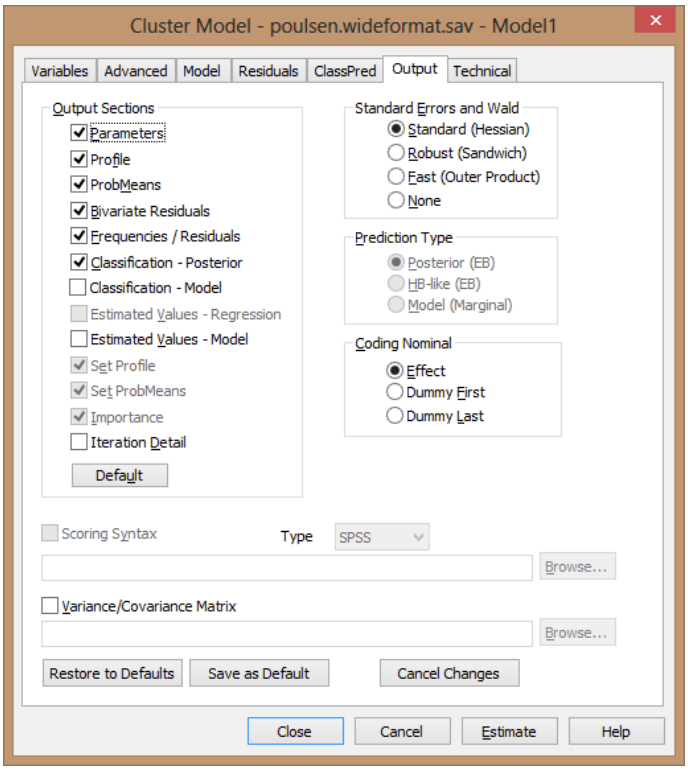

Figure 24. Output tab.

After estimating, name the models for future reference:

- Click on Model1 to highlight it, and then click on it again to enable the model renaming feature.
- Rename Model 1 '1-cluster' and Model2 '2-cluster'.

Then display the Profile plots for both models:

- click on Profile
- click on Profile plot

The Profile output for the 2-cluster model is shown in Figure 25. For the Profile plots, recall Figure 2.

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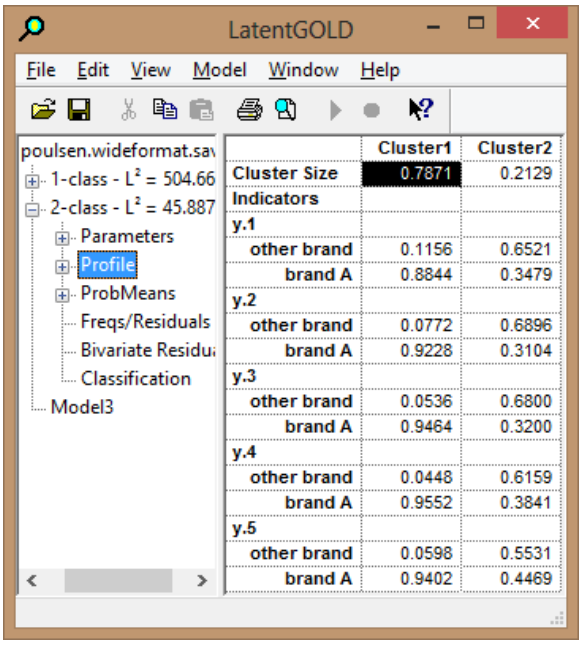

Figure 25. Profile for 2-cluster model.

- Click on the data file name to display the overall model fit

Since these data are not sparse, the L^2 fit statistic has a chi-squared distribution and thus the reported p-values are valid.

- Right click and use the Model Summary Display to remove LL, and BIC(LL) and add BIC(L^2).

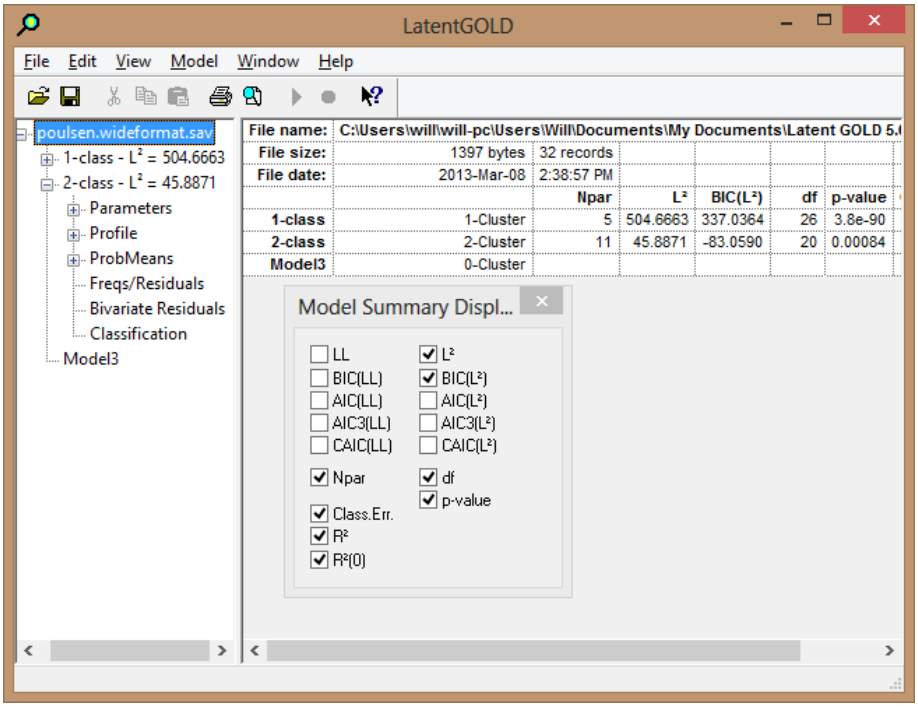

Figure 26. Model Summary output and Model Summary Display.

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*Note: Comparing Figure 26 with Figure 12 we see a slight difference in L^2 and BIC for the 2-class model estimated with the regression model (with long format) vs. cluster model (with wide format in Figure 26) due to slightly different way that Bayes constant is implemented in these modules. For all practical purposes these 2 approaches yield equivalent models (i.e., p-values are identical to 5 decimal places).

From column 'Npar' (Figure 26) we see that there are 11 – 5 = 6 additional parameters in the 2-class model (1 parameter for the size of class 2 plus the 5 additional purchase probabilities associated with this class). Although the 2-class latent growth model fits much better than the independence model, p-value = .00084 means that this model also fails to explain all the autocorrelation in the data.

Bivariate residuals (BVRs) can be used to test the significance of the autocorrelations remaining in the data, each BVR > 3.84 indicating significance at the .05 level. The BVRs under the independence model (Figure 27) indicate that all of the autocorrelations in these data are highly significant and follow a particular pattern, with the BVRs associated with 1st order correlations (adjacent time points highlighted in Figure 27) being largest.

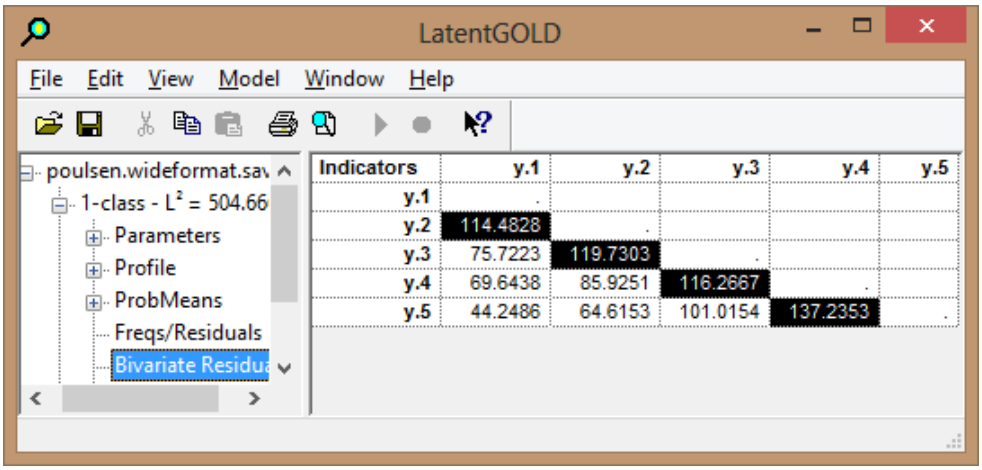

Figure 27. A large BVR means that the 1-class (null) model fails to explain the associated correlation.

Although the BVRs are substantially reduced by the 2-class model, two large BVRs exist (Figure 28) suggesting that the 1st order autocorrelation (especially for responses at adjacent times 1 and 2, as well as 4 and 5) remains unexplained.

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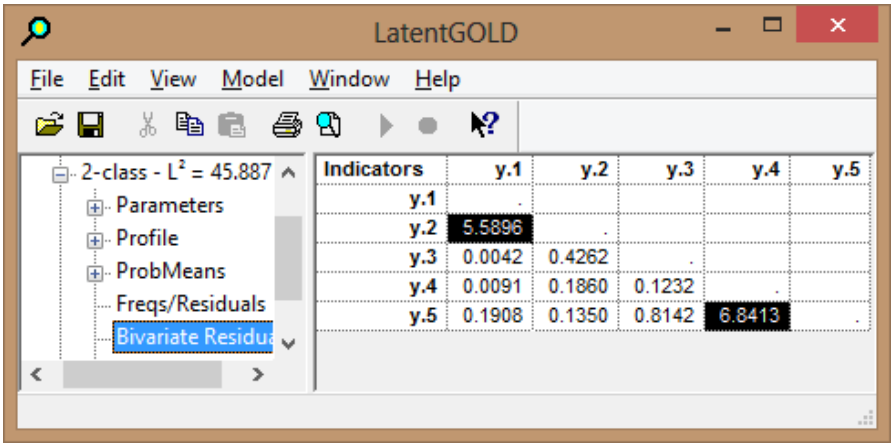
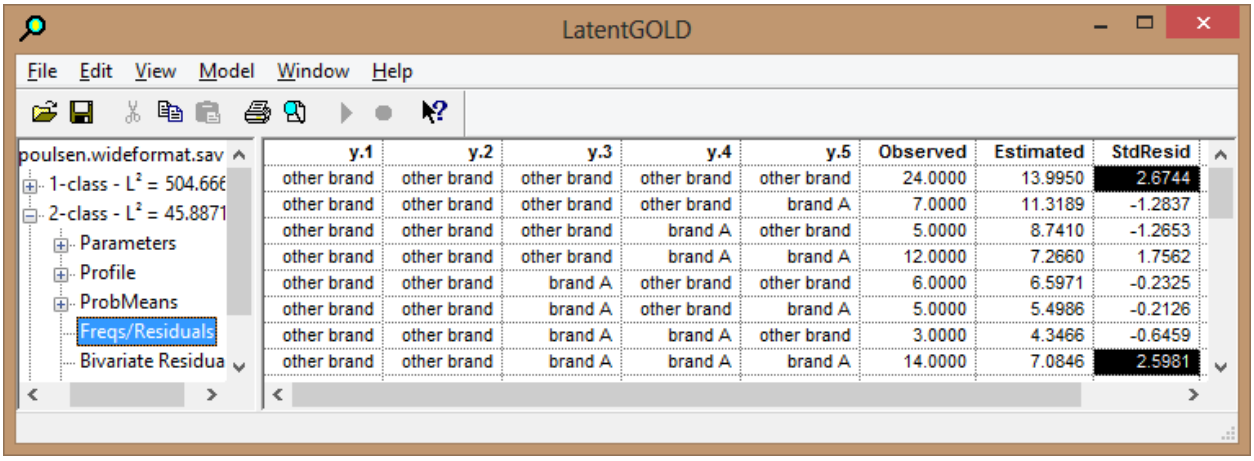


Indicators	y.1	y.2	y.3	y.4	y.5
y.1	.				
y.2	5.5896	.			
y.3	0.0042	0.4262	.		
y.4	0.0091	0.1860	0.1232	.	
y.5	0.1908	0.1350	0.8142	6.8413	.

Figure 28. Large BVRs indicate that 2-class model fails to explain all the autocorrelations.

Also, some large residuals (Figure 29 highlights StdResid > 2) support the failure of the 2-class latent growth model to adequately fit these data.



y.1	y.2	y.3	y.4	y.5	Observed	Estimated	StdResid
other brand	other brand	other brand	other brand	other brand	24.0000	13.9950	2.6744
other brand	other brand	other brand	other brand	brand A	7.0000	11.3189	-1.2837
other brand	other brand	other brand	brand A	other brand	5.0000	8.7410	-1.2653
other brand	other brand	other brand	brand A	brand A	12.0000	7.2660	1.7562
other brand	other brand	brand A	other brand	other brand	6.0000	6.5971	-0.2325
other brand	other brand	brand A	other brand	brand A	5.0000	5.4986	-0.2126
other brand	other brand	brand A	brand A	other brand	3.0000	4.3466	-0.6459
other brand	other brand	brand A	brand A	brand A	14.0000	7.0846	2.5981

Figure 29. 2-class frequencies and residuals output.