

## Choice Tutorial 12: Using Latent GOLD choice to Estimate Random Regret Minimization-based Discrete Choice Models

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In this tutorial you will learn to estimate Random Regret Minimization models. Specifically, in this tutorial you will:

- Estimate a RRM2010 model and interpret its results
- Estimate a  $\mu$ RRM model and interpret its results
- Estimate Latent class model comprising of two or more  $\mu$ RRM classes
- Explore which of these models provides the best fit to the data using the BIC statistic

### 1 The Data

Latent GOLD Choice accepts data from an optional 1-file or its default 3-file structure from an SPSS .sav file, a Sawtooth .cho file, or ASCII rectangular file format. The current sample data is in a SPSS save file named 'Shopping\_data\_LG.sav.'

The data contain 1,503 Revealed Preference (RP) shopping location choices from 1,074 consumers who went shopping with the aim to buy groceries (see Arentze et al. 2005 for more details). Each alternative is defined by three attributes: 'Floor Space Groceries' (FSG), 'Floor Space Other' (FSO), and Travel Time (TT) to reach the shopping location. Choice sets are imputed to contain the chosen alternative and 4 non-chosen alternatives.

Table 1 lists the variables in the data set. Furthermore, Figure 1 provides histograms showing the distribution of the three attribute levels:

<i>Variable</i>	<i>Explanation</i>
CASEID	Identifier of the respondent
SETID	Identifier of the choice set
CHOICE15	Choice
Alternative	Identifier of the alternative in the choice set
FSG	Floor Space Groceries
FSO	Floor Space Other
TT	Travel Time
CHOICE01	Choice (dichotomous)

Table 1: Description of the data set

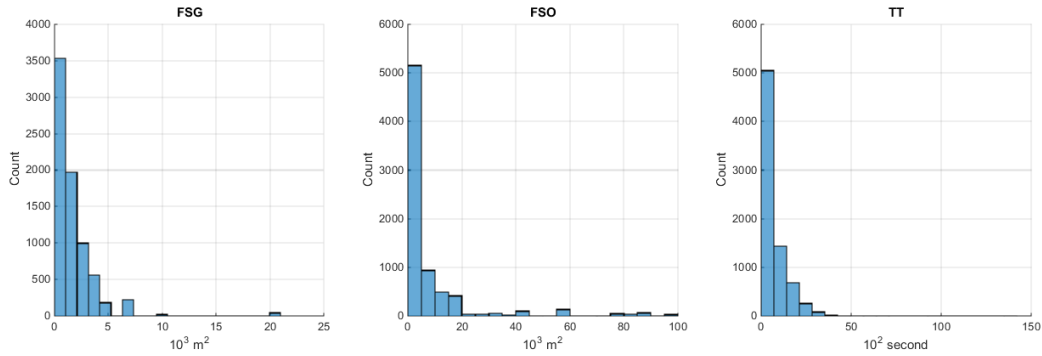


Figure 1: Attribute level distribution

## 2 The Goal

The goal is develop insights on the shopping location choice behaviour of the respondents. That is, we wish to acquire understanding on how the respondents traded-off FSG, FSO and TT.

To do so, first we estimate discrete choice models based on different decision rules. Specifically, we estimate a RRM2010 model (Chorus 2010) and the recently proposed more flexible  $\mu$ RRM model (Van Cranenburgh et al. 2015). Furthermore, decision rule heterogeneity may be present. That is, not all respondents may adopt the same decision rule; rather, different respondents may adopt different decision rules when making shopping location choices. For instance, the choice of one class of respondents may be more consistent with a utility maximization decision rule, while the choices of another class may be more consistent with regret minimization decision rules. We therefore estimate LC class models, where the classes represent different decision rules. Finally, we assess which model best describes the choice processes in the data using the BIC statistic.

## 3 Setting up the analysis

- First, open the data file: File → Open → browse to the working directory
- Select 'Shopping\_data\_LG.sav'
- Right-click on Model1, and choose 'Choice'. The Model Analysis Dialog Box opens (Figure 2)



Figure 2: Model Analysis Dialog Box

- Select each variable and move it to the appropriate box by clicking the buttons to the left of these boxes. That is:
  - Move CHOICE01 to Dependent
  - Move CASEID to Case ID
  - Move SETID to Choice Set
- Tick '1 File' to indicate that all the data can be found in a single data file. See the LG User Guide for more details on the data file and formats (Vermunt and Magidson 2005).
- Open the Attributes tab by clicking on 'Attributes' at the top of the setup screen. The Attributes Dialog Box opens (Figure 3).

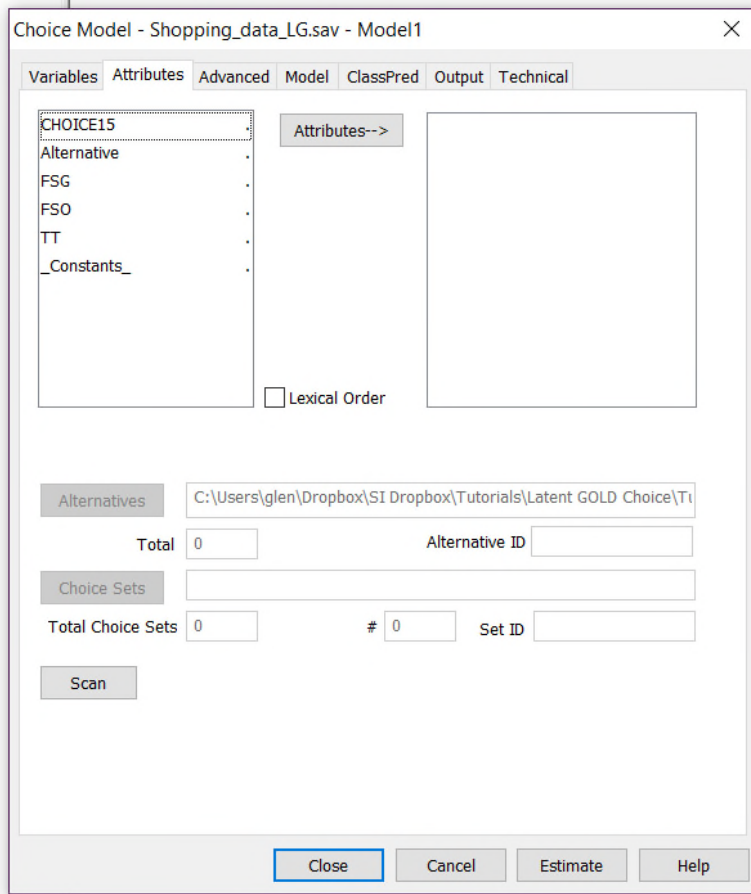


Figure 3: Attributes Dialog Box

- Select each attribute (FSG, FSO, and TT) and move them to the Attribute list box by clicking the buttons to the left of these boxes.
  - Move FSG to the Attributes box
  - Move FSO to the Attributes box
  - Move TT to the Attributes box

#### 4 Single class models

Now that we have specified the data for analysis, we are ready to estimate discrete choice models. We first will estimate a (1-Class) linear-additive RUM model. This model serves as the baseline model. After that, we estimate a (1-Class) RRM2010 model, and a (1-Class)  $\mu$ RRM model. And finally, we will estimate Latent Class models.

- In the 'Variables' Tab set the number of classes to '1'

#### 4.1 Estimating a linear-additive RUM model

By default, Latent Gold Choice estimates a model using a linear-additive RUM specification.

- Click Estimate (located at the bottom right of the analysis dialog box).

The setup window now closes and LG starts the estimations. When Latent GOLD Choice completes the estimation a list of various output files appears in the Content pane, see Figure 4 below where Model 1 is associated with  $L^2 = 4610.4938$ .

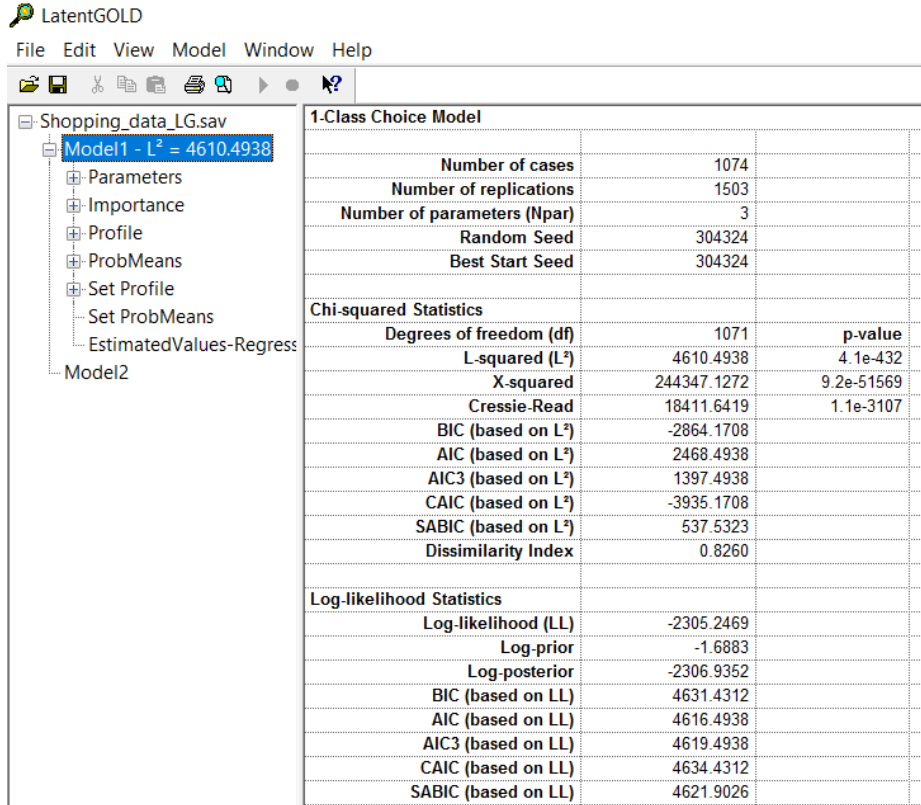


Figure 4: Estimation Results in Content Pane

#### 4.2 Estimating a RRM2010 model

Random regret minimization models postulate that decision makers, when choosing, experience regret when a competitor alternative  $j$  outperforms the considered alternative  $i$  with regard to one or more attributes  $m$ . The overall regret of an alternative is conceived to be the sum of all the pairwise regrets that are associated with bilaterally comparing the considered alternative with the other alternatives in the choice set. Decision makers are assumed to choose the minimum regret alternative (see Chorus 2012 for a more detailed explanation).

The mathematical form of RRM models is given in Equation 1, where  $RR_{in}$  denotes the random regret for decision maker  $n$  considering alternative  $i$ ,  $R_{in}$  denotes the observed part of regret, and  $\varepsilon_{in}$  denotes the unobserved part of regret.

$RR_{in} = R_{in} + \varepsilon_{in} \text{ where } R_{in} = \sum_{j \neq i} \sum_m r_{ijmn}$	<b>Equation 1</b>
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In the core of RRM models is the so-called attribute level regret function:

$r_{ijmn} = f(\beta_m, x_{jmn} - x_{imn})$ . This function maps the difference between the attribute levels of attributes  $m$  of the competitor alternatives  $j$  and the considered alternative  $i$  onto regret. Note that in our notation  $\beta_m$  denotes the taste parameter (to be estimated) associated with attribute  $m$  (e.g. travel time) and  $x_{jmn}$ ,  $x_{imn}$  denote, respectively, the attribute level of alternative  $j$  and  $i$  for attribute  $m$ . Different specifications of the attribute level regret function lead to different types of random regret models (see Van Cranenburgh and Prato 2016 for an overview).

The attribute level regret function of the RRM2010 model proposed by Chorus (2010) is depicted in Figure 5.

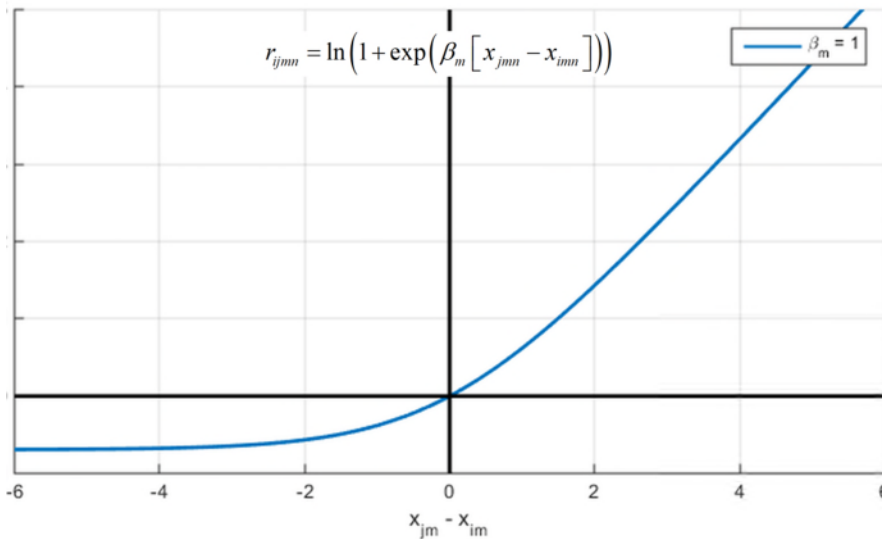
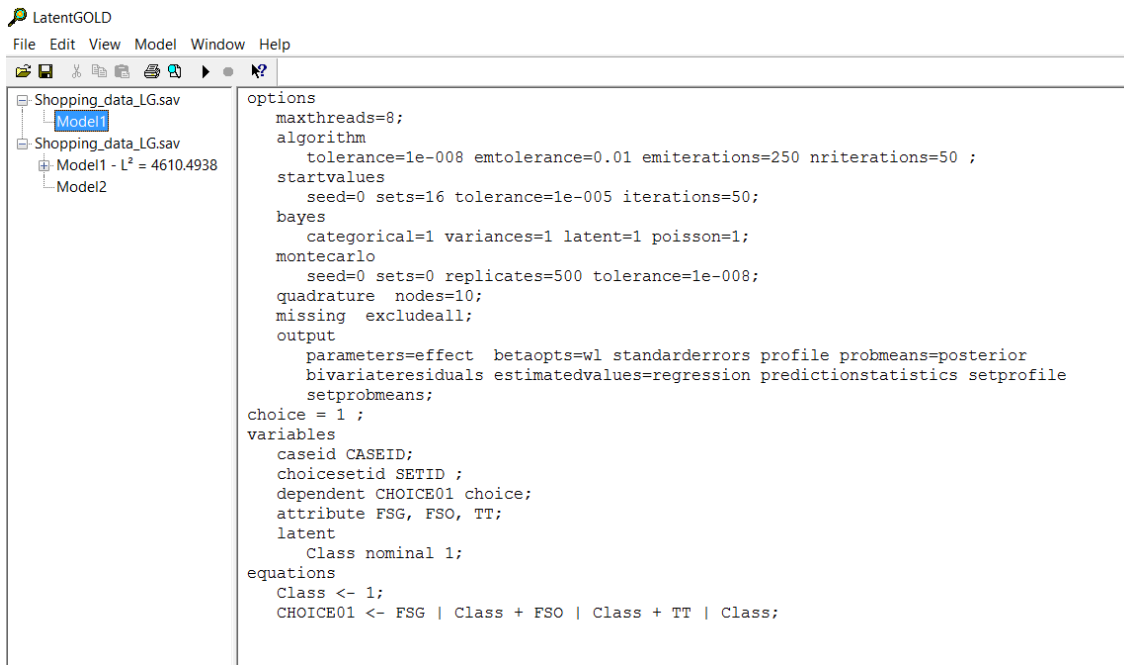


Figure 5: Attribute level regret function RRM2010

To estimate this model in LG Choice, we need to use the LG syntax. This can most easily be achieved by modifying the syntax of the linear-additive RUM model.

- Right Click ‘Model1’ in Outline Pane. A list box will appear.

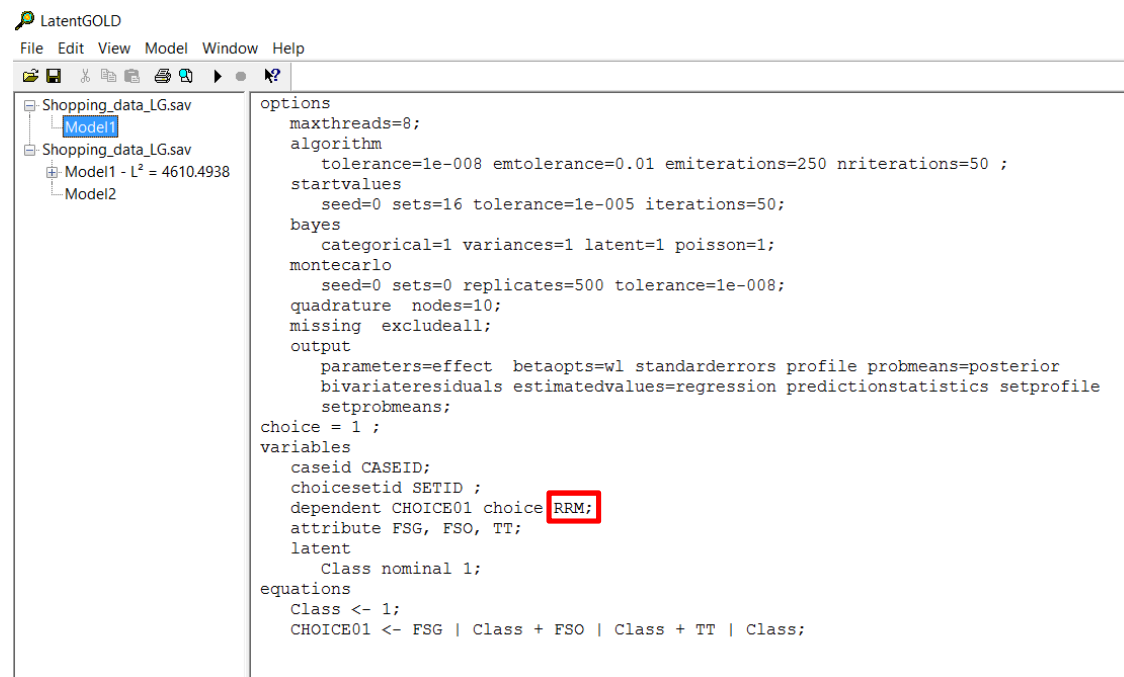
- Left Click on ‘Generate Syntax’. The LG syntax Dialog box will appear in the Content Pane, see Figure 6.



```
options
maxthreads=8;
algorithm
  tolerance=1e-008 emtolerance=0.01 emiterations=250 nriterations=50 ;
startvalues
  seed=0 sets=16 tolerance=1e-005 iterations=50;
bayes
  categorical=1 variances=1 latent=1 poisson=1;
montecarlo
  seed=0 sets=0 replicates=500 tolerance=1e-008;
quadrature nodes=10;
missing excludeall;
output
  parameters=effect betaopts=wl standarderrors profile probmeans=posterior
  bivariateresiduals estimatedvalues=regression predictionstatistics setprofile
  setprobmeans;
choice = 1 ;
variables
  caseid CASEID;
  choicesetid SETID ;
  dependent CHOICE01 choice;
  attribute FSG, FSO, TT;
  latent
    Class nominal 1;
equations
  Class <- 1;
  CHOICE01 <- FSG | Class + FSO | Class + TT | Class;
```

Figure 6: LG Choice syntax Dialog Box

- Now we change the syntax to tell LG to estimate a RRM2010 model. We add the syntax keyword ‘RRM’ to the right of the keyword ‘choice’, see Figure 7 below.



```
options
maxthreads=8;
algorithm
  tolerance=1e-008 emtolerance=0.01 emiterations=250 nriterations=50 ;
startvalues
  seed=0 sets=16 tolerance=1e-005 iterations=50;
bayes
  categorical=1 variances=1 latent=1 poisson=1;
montecarlo
  seed=0 sets=0 replicates=500 tolerance=1e-008;
quadrature nodes=10;
missing excludeall;
output
  parameters=effect betaopts=wl standarderrors profile probmeans=posterior
  bivariateresiduals estimatedvalues=regression predictionstatistics setprofile
  setprobmeans;
choice = 1 ;
variables
  caseid CASEID;
  choicesetid SETID ;
  dependent CHOICE01 choice RRM;
  attribute FSG, FSO, TT;
  latent
    Class nominal 1;
equations
  Class <- 1;
  CHOICE01 <- FSG | Class + FSO | Class + TT | Class;
```

Figure 7: LG Choice syntax Dialog Box

- Click the Estimation Button ► at the top of the setup screen.

The setup window now closes and LG starts estimating the RRM2010 model. When LG Choice completes the estimation a list of various output files appears in the Content pane, see Figure 8 below. Confirm that you again get  $L^2 = 4610.4938$  for this model.

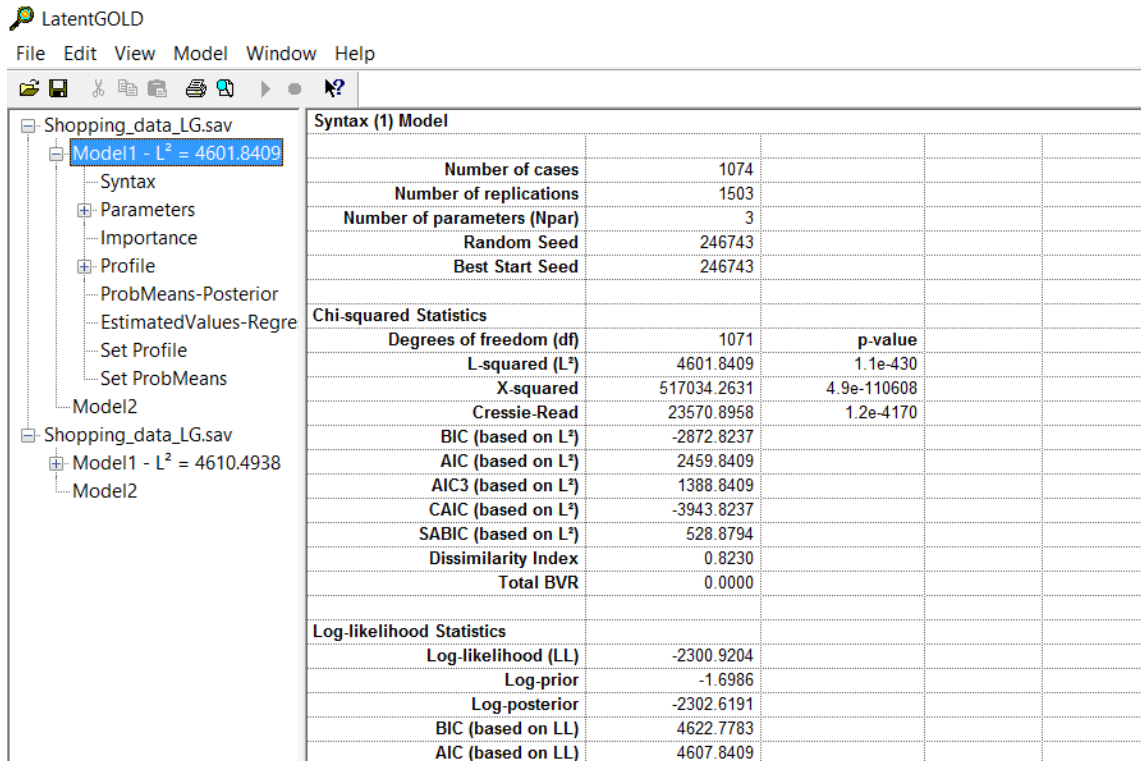


Figure 8: RRM2010 Estimation Results in Content Pane

### 4.3 Estimating a $\mu$ RRM model

The  $\mu$ RRM model proposed by Van Cranenburgh et al. (2015) generalizes the RRM2010 model by allowing the scale parameter  $\mu$  to be estimated. The  $\mu$ RRM model has three special cases: 1) the RRM2010 model, 2) the linear-additive RUM model, and 3) the P-RRM model. See Van Cranenburgh et al. (2015) for a full description.

The attribute level regret function of the  $\mu$ RRM model is depicted in Figure 9. As can be seen, the shape of the attribute level regret function – and hence the degree of regret minimization behaviour the model imposes – varies with the size of the scale parameter  $\mu$ . In case  $\mu$  is large, the attribute level regret function is approximately linear (purple line). This implies that the attributes are traded-off in a compensatory way (just like the linear-additive RUM model does). In case  $\mu$  is small, the attribute level regret function is



highly asymmetric (orange line). In that case, trade-offs are made in a highly non-compensatory way.

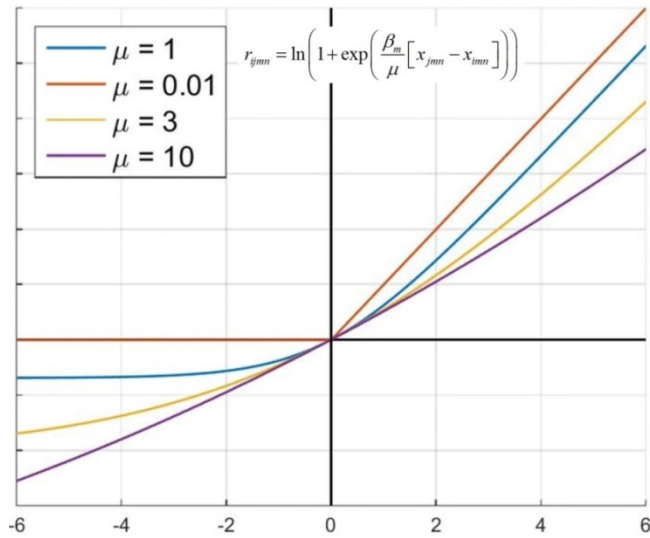


Figure 9: Shapes of the Attribute level regret function of the  $\mu$ RRM model

- To estimate the  $\mu$ RRM model we need to change the syntax again. Click on ‘Model2’ in the outline pane to bring up the syntax in the contents pane on the right.
- To tell LG to estimate a scale parameter, we add the following syntax: ‘CHOICE01 <<- 1;’, see Figure 10.

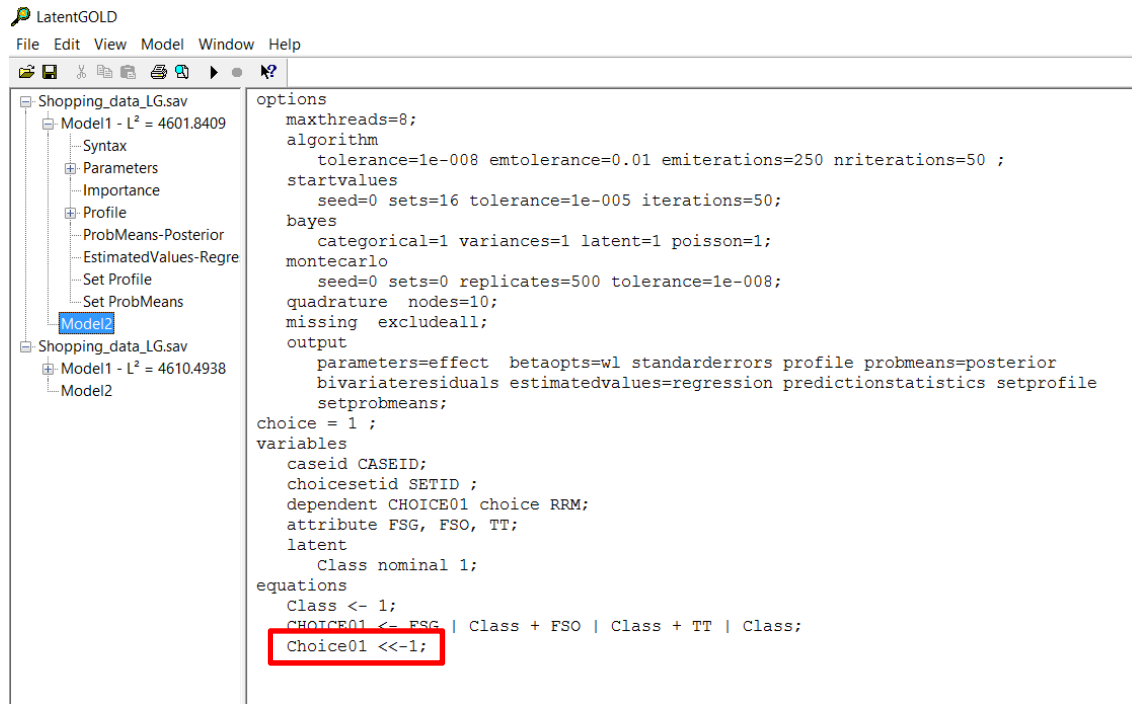


Figure 10: LG Choice syntax Dialog Box

- Click the Estimation Button ► at the top of the setup screen.

The setup window now closes and LG starts estimating the  $\mu$ RRM model. When Latent GOLD Choice completes the estimation a list of various output files appears in the Content panel, see below.

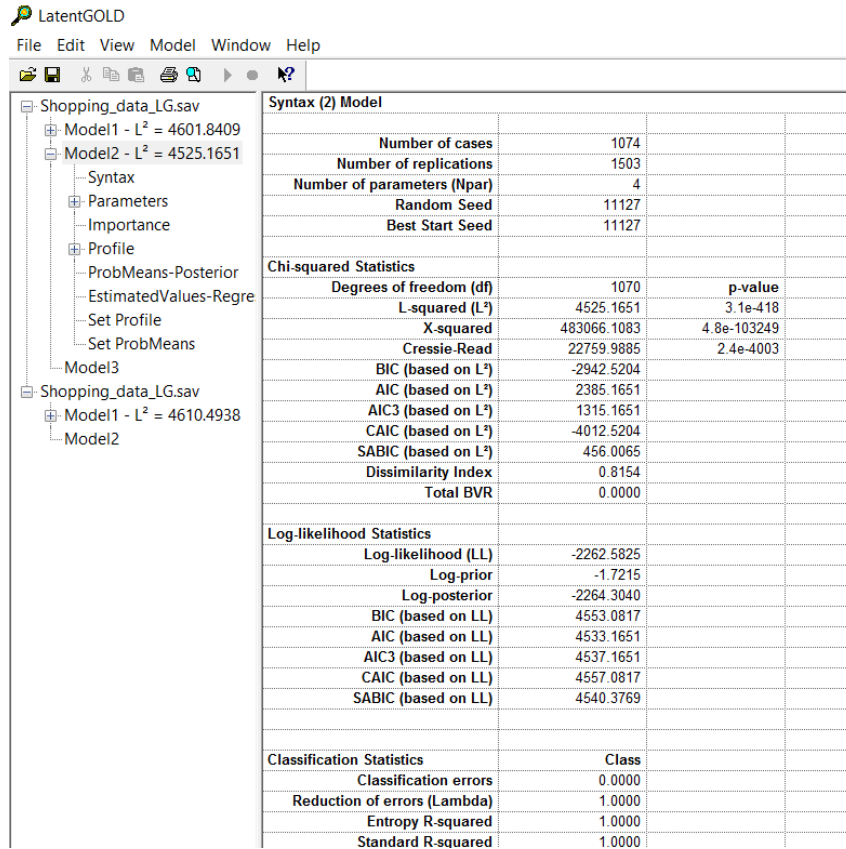


Figure 11:  $\mu$ RRM Estimation Results in Content Pane

#### 4.4 Interpreting Results

To assess which model (RUM, RRM2010,  $\mu$ RRM) provides the best fit to the data we use test statistics. Since the RRM and RUM model are not special cases of one another we cannot use the commonly used Likelihood Ratio Statistic (LRS) to assess whether one model describes the data significantly better than the other. Instead, we need to use a test suitable for non-nested models. The Ben-Akiva and Swait (BS) test (see the textbox below) is one such test (Ben-Akiva and Swait 1986). When comparing the  $\mu$ RRM model with the RUM and the RRM2010 models we can however use the LRS (Equation 2) as the  $\mu$ RRM model nests both the linear-additive RUM model and the RRM2010 model. The LRS is  $\chi^2$  distributed with  $k$  degrees of freedom.

The BS test gives an upper bound for the probability that, when some model A achieves a lower log-likelihood than some other (non-nested) model B, A is still the correct model of the data-generating process. This upper bound can therefore be considered a conservative proxy for the significance (or: p-value) of a difference in model fit between two non-nested models A and B, see Chorus (2012).

The BS statistic is given in the equation below:

$$\Pr[\rho_A^2 - \rho_B^2 \geq z] \leq \Phi\left[-(2Nz \ln J + (K_A - K_B))^{1/2}\right] \text{ where,}$$

$$\rho^2 = 1 - \frac{LL(\hat{\beta}) - K/2}{LL(0)}$$

$K$  is the number of model parameters

$z$  is the model fit difference

$N$  is the number of observations

$J$  is the choice set size

$K$  is the number of parameters

$\Phi[ ]$  is the standard normal cumulative distribution function

$LRS = -2(LL_{Model A} - LL_{Model B})$	<a href="#">Equation 2</a>
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- Click on ‘Shopping\_data\_LG.sav’ in the Outline Pane. A summary of all the models estimated on this data file appears in the Content Pane, see Figure 12.

Looking at the Final-Loglikelihoods (LL) we see that the RRM2010 performs slightly better than the RUM model (about 4.3 log-likelihood points). However, applying the BS test (Equation 3 reveals that the RRM2010 describes the data somewhat better.

$\Pr\left[\frac{LL_B(\hat{\beta}) - LL_A(\hat{\beta})}{LL(0)} \geq z\right] \leq \Phi\left[-(2Nz \ln J + (K_A - K_B))^{1/2}\right]$ $\Pr[1.78 \cdot 10^{-3} \geq z] \leq \Phi\left[-(2 \cdot 1504 \cdot 1.78 \cdot 10^{-3} \ln 5)^{1/2}\right]$ $\Pr[1.78 \cdot 10^{-3} \geq z] \leq 0.002$	<a href="#">Equation 3</a>
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To see whether the  $\mu$ RRM model outperforms its special cases in the statistical sense, we apply the LRS. The LRS (Equation 4) show that the  $\mu$ RRM model describes the data (highly) significantly better than its special cases. The LRS far exceeds the critical  $\chi^2$  value of 3.84 with 1 degree of freedom at a significance level of  $\alpha = 0.05$ . An alternative statistic to assess which model fits statically best is the Bayesian Information Criterion (BIC). This criterion is not so commonly used in the field of choice modelling, other than in the context of Latent Class discrete

choice model applications. However, in this case, the BIC correctly identifies the  $\mu$ RRM model as the best model (lowest BIC).

$LRS = -2(LL_{RUM} - LL_{\mu RRM})$ $LRS = -2(-2305.247 - -2262.583) = 85.33$ $LRS = -2(LL_{RRM2010} - LL_{\mu RRM})$ $LRS = -2(-2300.920 - -2262.583) = 76.67$	<b>Equation 4</b>
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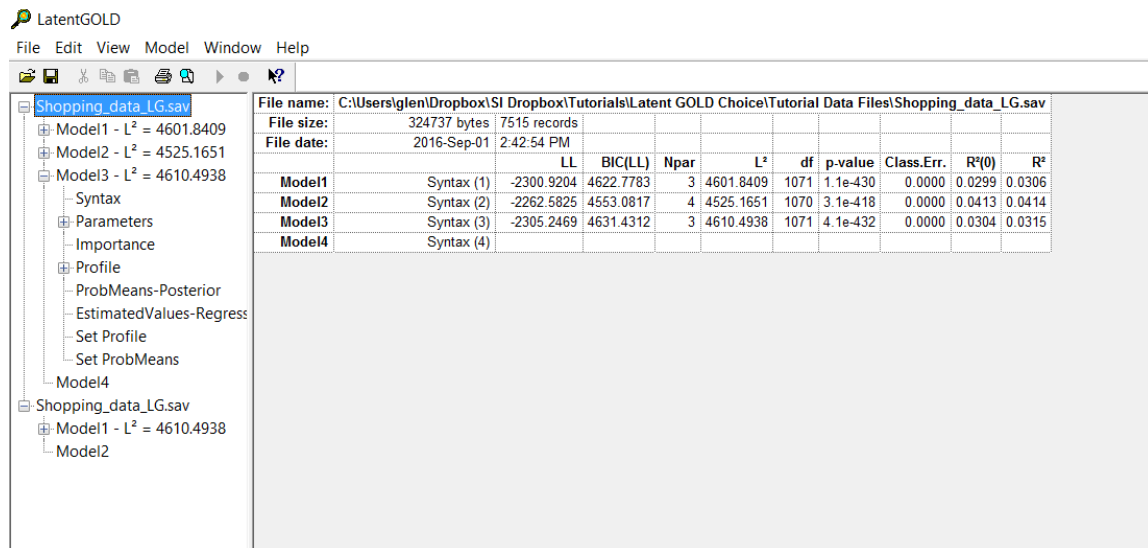


Figure 12: Model Summary Output

## 4.5 Examining the Model Output files

### 4.5.1 RRM2010

First we look at the results for the RRM2010 model.

- Click on the expand icon (+) next to the RRM2010 model. Several output files appear.

To view whether the parameter estimates are significantly different from 0:

- Click on Parameters to view the part-worth utility estimates in the Contents Pane
- Right click on the ‘View’ and select ‘Standard Errors’ and ‘Z Statistic’ from the dropdown menu (Figure 13).

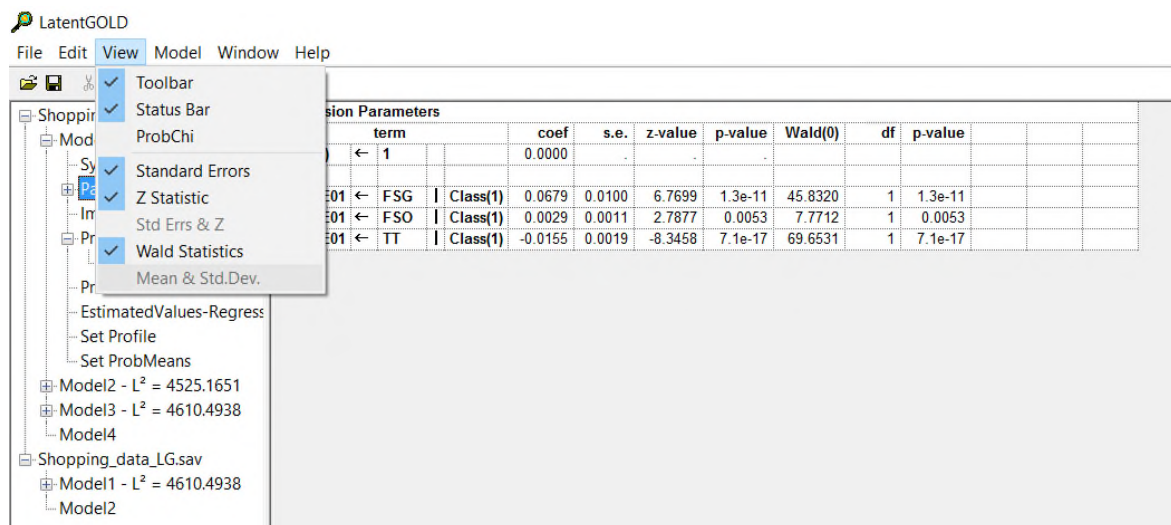


Figure 13: Viewing Standard Error, Z statistic and associated p-values

The parameter estimates and the associated standard errors, Z-values and p-values in Figure 13 show that all 3 attributes (FSG, FSO and TT) are significantly different from 0.

To learn about the degree of regret minimizing behaviour imposed by RRM models we may compute the so-called ‘profundity of regret measure’, see Van Cranenburgh et al. (2015). This measure is computed for each attribute. A profundity of regret of close to 1 indicates that a very strong degree of regret minimization behaviour is associated with that attribute, while a profundity of regret close to 0 indicates that a very mild degree of regret minimization behaviour is associated with that attribute. In other words, a profundity of regret close to 1 signals highly non-compensatory regret minimization behaviour, whereas a profundity of regret close to 0 signals compensatory behaviour. In fact, when the profundity of regret equals 0, the attribute is

processed in a linear-additive RUM way. Software code to compute this measure can we found at [www.advancedrrmmodels.com](http://www.advancedrrmmodels.com)

Table 2 shows the profundity of regret levels for the RRM2010 model. It shows that all profundity of regrets are close to zero. Hence, the behaviour imposed by the RMM2010 is close to compensatory RUM behaviour. As such, it comes as no surprise that the model fit difference between the linear-additive RUM model (LL = -2305.25) and the RRM2010 (LL = -2300.92) is quite small.

Attribute	Profundity of regret
FSG	0.06
FSO	0.02
TT	0.06

**Table 2: Profundity of regret RRM2010**

### 4.5.2 $\mu$ RRM

Next, we look at the results for the  $\mu$ RRM model.

- Click on the expand icon (+) next to the  $\mu$ RRM model. Several output files appear.
- To view the parameter estimates Click on Parameters to view the part-worth utility estimates in the Contents Pane
- Right click on ‘View’ and select ‘Standard Errors’ and ‘Z Statistic’ from the dropdown menu, see Figure 14

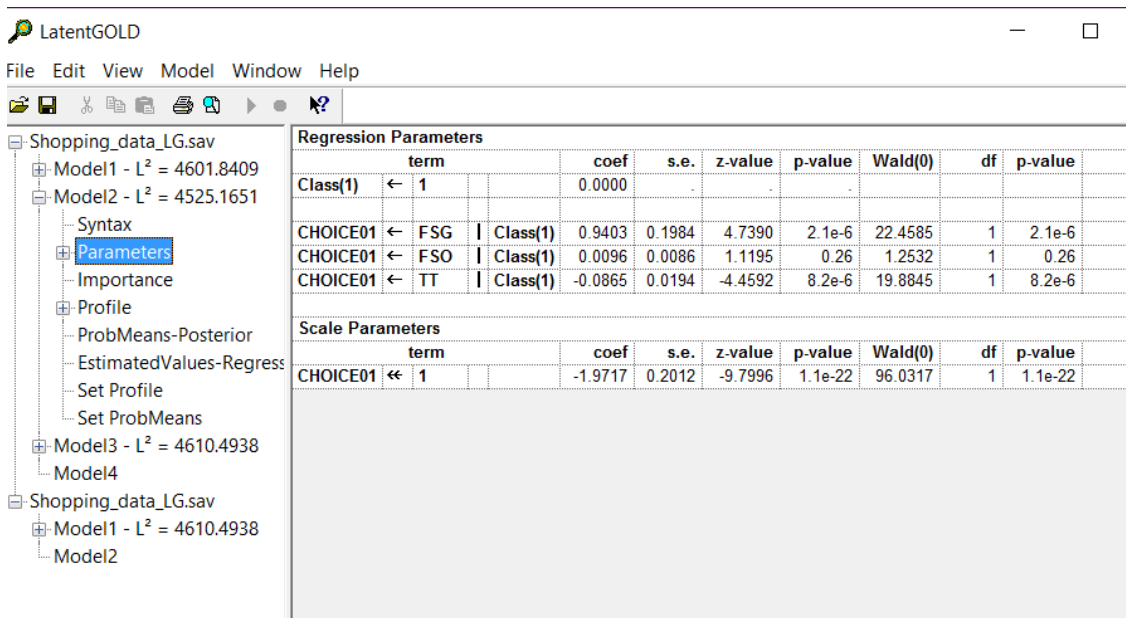


Figure 14:  $\mu$ RRM results

Notice that the parameter estimates and the associated standard errors, Z-values and p-values in Figure 14 show that FSG and TT are significantly different from 0, while – in contrast to the results found using the RRM2010 model – FSO is no longer significantly different from zero at a significance level of  $\alpha = 0.05$ .

Importantly, the parametrization of the  $\mu$ RRM model differs from the ‘standard parametrization’ proposed by Van Cranenburgh et al. (2015). Equation 5 shows the parametrization of the  $\mu$  RRM model in LG Choice; Equation 6 shows the standard parametrization.

$R_i = \sum_{j \neq i} \sum_m \left( 1 + \exp \left( \beta_m^* [x_{jm} - x_{im}] \right) \right) \quad P_i = \frac{e^{-e^{\mu^*} R_i}}{\sum_j e^{-e^{\mu^*} R_j}} \quad (\text{LG parametrization})$	<b>Equation 5</b>
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$R_i = \sum_{j \neq i} \sum_m \left( 1 + \exp \left( \frac{\beta_m}{\mu} [x_{jm} - x_{im}] \right) \right) P_i = \frac{e^{-\mu R_i}}{\sum_j e^{-\mu R_j}} \quad (\text{Standard parametrization})$	<b>Equation 6</b>
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Equation 5 and Equation 6 differ from one another in two important ways:

1. The taste parameter  $\beta_m^*$  is not divided by the scale parameter  $\mu$  in the attribute level regret function:
2. In LG Choice the log of the scale  $\mu$  is estimated, instead of the scale (for numerical reasons).

These differences in parametrization have several implications. Firstly, it means that the scale parameter reported by LG needs to be exponentiated (see Equation 7, where \* denotes the LG scale parameter estimate) to obtain the scale  $\mu$ . Secondly, to cross-validate or compare results of LG Choice with results obtained using the standard parametrization requires some additional steps. As the parameters are transformed, standard errors are likely to be different across parametrizations. These steps however go beyond the scope of this tutorial. Interested readers are referred to (Van Cranenburgh and Vermunt 2015). Finally, it is worthwhile to mention that although the parametrizations are different, these models are equivalent -- both impose the same behavioural restrictions and hence result in the same model fit.

$\mu = e^{\mu^*}$	<b>Equation 7</b>
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Accordingly, the scale parameter is found to be  $\mu = e^{-1.9717} = 0.14$

Table 3 shows the profundity of regret levels for the  $\mu$ RRM model. It shows that substantially regret minimizing behaviour is present for FSG and TT. The strong deviation from compensatory behaviour explains the substantial difference in model fit between the linear-additive RUM model (LL = -2305.25) and the  $\mu$ RRM model (LL = -2262.58).

Attribute	Profundity of regret
FSG	0.50
FSO	0.05
TT	0.32

**Table 3: Profundity of regret  $\mu$ RRM**

## 5 Latent Class Random Regret Minimization models

In this section we are going to estimate Latent Class models which consist of multiple  $\mu$ RRM models.

- In the syntax, set the number of Latent classes to '2', see Figure 15.
- To tell LG to allow for differences in scale (hence different decision rules) across classes, also add the '| Class;', see Figure 15.

Latent Class models can get stuck in local maxima during the estimation. To deal with this the user can change the estimation algorithm settings using the syntax, such as the seed number, the number of starting value sets, and the number of EM iterations per set.

- To reduce the probability on finding a local solution, increase the number of EM iterations from 50 to 250 in the syntax, see Figure 15.

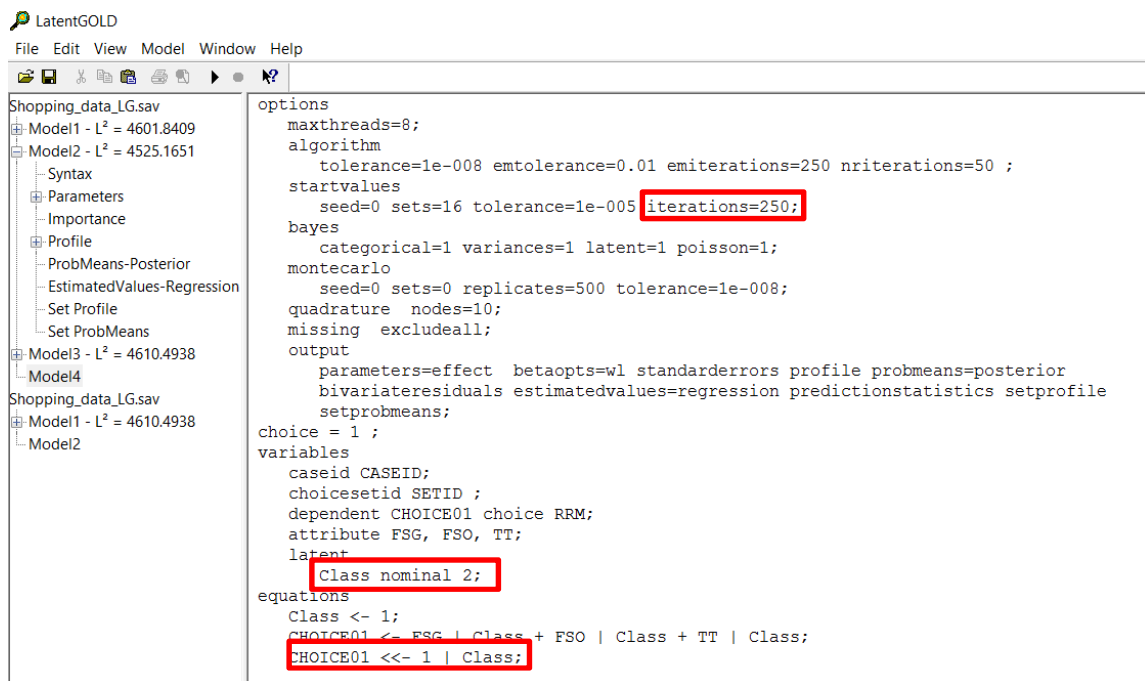


Figure 15: LG Choice syntax Dialog Box

Notice that since the shopping choice data set does not contain explanatory variables that can be used to explain class membership, only class-specific constants are estimated. Therefore, formally the Latent Class models we estimate are discrete mixture models.

- Click the Estimation Button ► at the top of the setup screen to estimate the 2- $\mu$ RRM class model.

Next, we estimate a 3- $\mu$ RRM class Latent class model.

- Navigate in the Outline Pane to Model15

- In the syntax, set the number of latent classes to '3', see Figure 15.
- Click the Estimation Button ► at the top of the setup screen to estimate the 3- $\mu$ RRM class model.
- You may click on the models in the outline pane at left to rename the models.

For completeness, we estimate also a 2-class and a 3-class RUM Latent class model.

- Navigate in the Outline Pane to Model6
- In the syntax, remove the 'RRM' command, see Figure 16 .

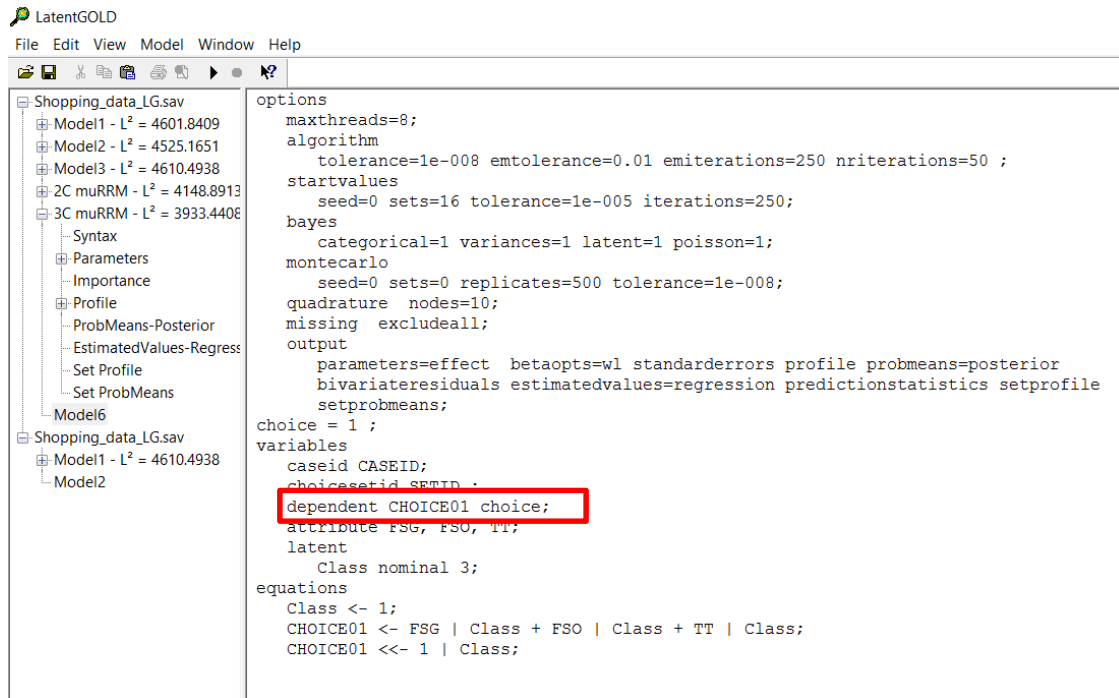


Figure 16: LG Choice syntax Dialog Box

- Click the Estimation Button ► at the top of the setup screen to estimate the 2-class RUM class model.

Notice that we still estimate the scale parameters. Hence, we allow for differences in error variance across classes. See Magidson and Vermunt (2007) for a discussion on the scale parameter in Latent Class choice models.

To estimate a 3-class RUM Latent class model

- Navigate to the syntax in the Outline Pane to Model7
- In the syntax, set the number of latent classes to ‘2’ and click the estimate button.

## 5.1 Interpreting Results

To assess which Latent Class model provides the best fit to the data we use the BIC statistic.

- Click on ‘Shopping\_data\_LG.sav’ in the Outline Pane. A summary of all the models estimated on this data file appears in the Content Pane, see Figure 17.

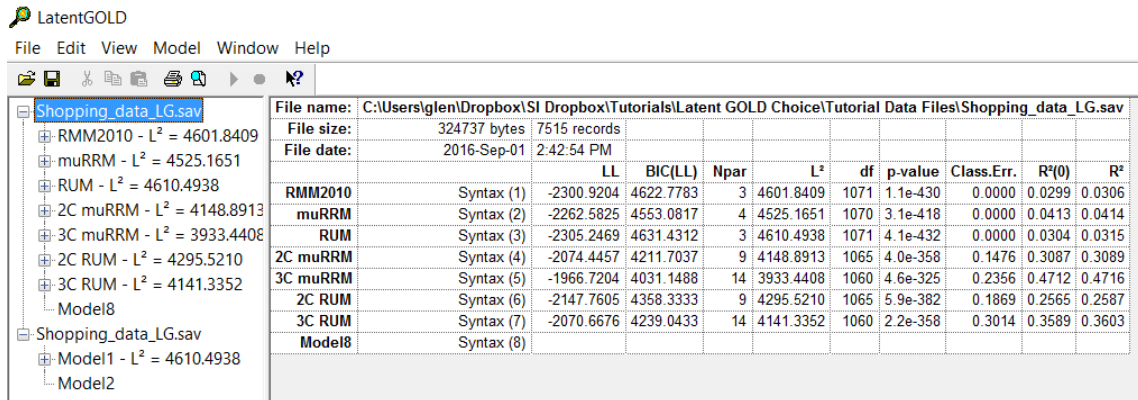


Figure 17: Model Summary Output

Based on the BIC statistic we infer that the 3- $\mu$ RRM class model (Model5) best fits the data. Note that the 3- $\mu$ RRM class model strongly outperforms the 3-class RUM model.

## 5.2 Examining the Model Output files

Figure 18 shows the estimated parameters for the best performing model: the 3- $\mu$ RRM class model.

Looking at the scale parameter estimates we see that classes 2 and 3 have roughly the same scale. This means that the differences between these classes is not so much related to decision rules, but rather to taste heterogeneity. This is also indicated by the Wald statistic, which shows that the scale parameters are not significantly different from one another.

Using Equation 7 we find the following scales of classes 1 to 3 (Table 4).

	$\mu = e^{\mu^*}$
Class 1	$\mu = e^{-1.3164} = 0.27$
Class 2	$\mu = e^{-2.1766} = 0.11$

Class 3	$\mu = e^{-2.2664} = 0.10$
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Table 4: Scale parameters for classes 1 to 3

Looking at the taste parameters in Figure 18, we see that FSG is significantly different from zero in all three classes. FSO and TT are however no longer significantly different from zero. This is likely to be the result of the limited number of observations, relative to the number of estimated parameters (19). As such, the 2-class model may be preferred for further analysis, despite the fact that the 3- $\mu$ RRM class model has a higher BIC value. Furthermore, the parameters of Class 2 are relatively small (close to zero). This signals that the choice consistency in this class is low. In other words, the choice behaviour in this class is relatively random. This can also be verified by looking at the choice probabilities of Class 2 under Set Profiles, which are all close to 20%.

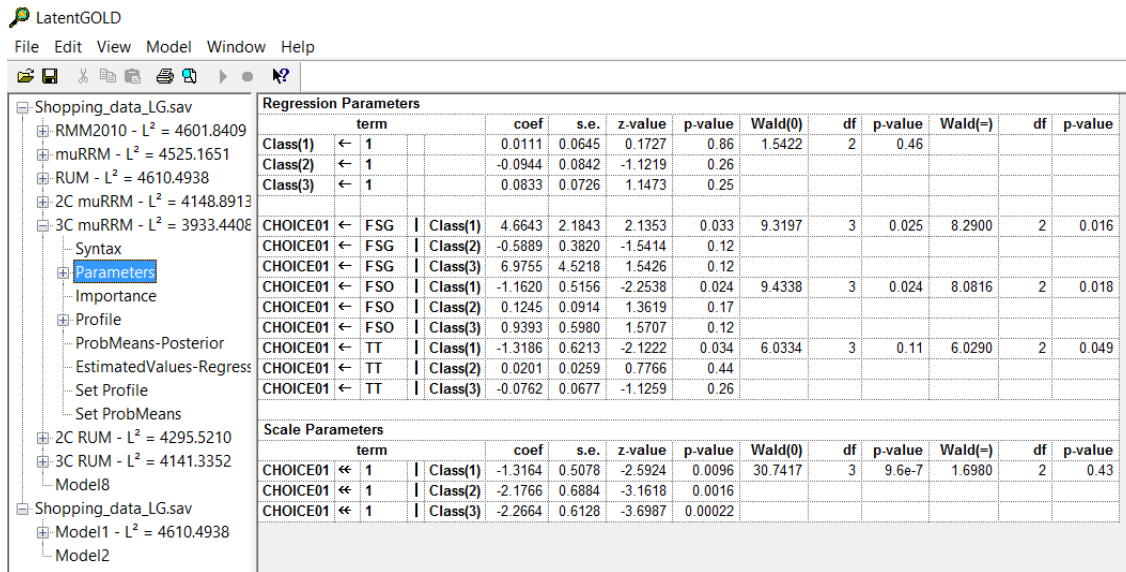


Figure 18: 3- $\mu$ RRM class results

Table 5 shows profundity of regret levels for the 3  $\mu$ RRM classes. It shows that most profundity of regrets are substantially different from zero. This indicates that in all classes substantial regret minimization behaviour is present.

Attribute	Profundity of regret		
	Class 1	Class 2	Class 3
FSG	0.844	0.367	0.891
FSO	0.802	0.361	0.361
TT	0.920	0.080	0.280

Table 5: Profundity of regret 3- $\mu$ RRM class model

Finally, we look at the class membership.

- Click in the Outline Pane on ProbMeans-Posterior to see the class membership probabilities, see Figure 19.

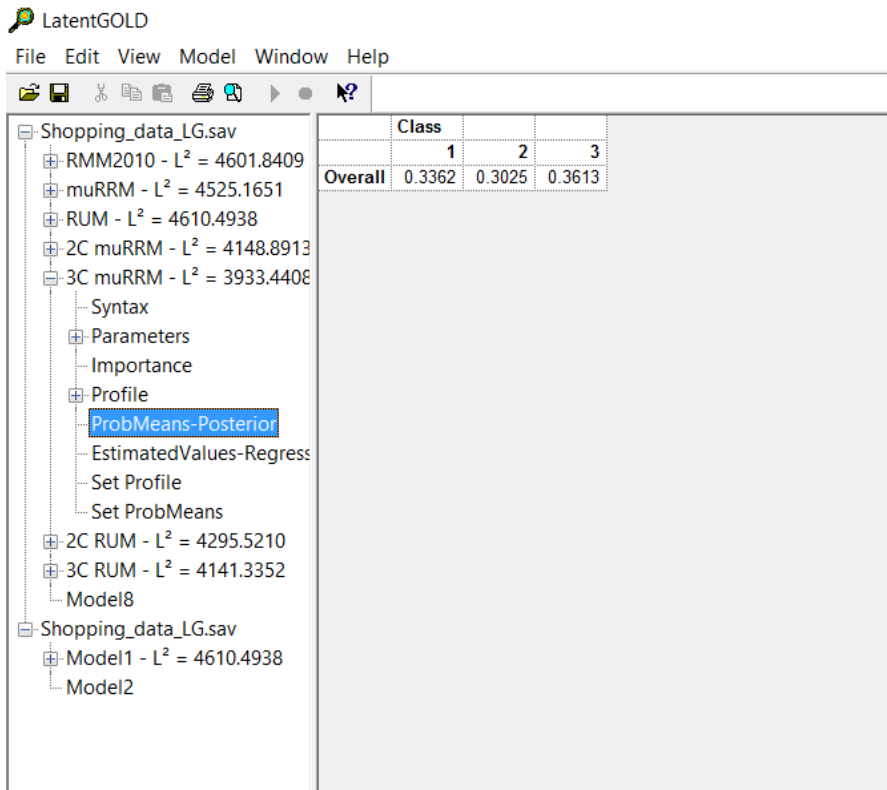


Figure 19: ProbMeans-Posterior

The results in Figure 19 show that the 3 classes are roughly of equal size.

## References

- Arentze, T. A., Oppewal, H. & Timmermans, H. J. P. (2005). A Multipurpose Shopping Trip Model to Assess Retail Agglomeration Effects. *Journal of Marketing Research*, 42(1), 109-115.
- Ben-Akiva, M. & Swait, J. (1986). The Akaike Likelihood Ratio Index. *Transportation Science*, 20(2), 133-136.
- Chorus, C. G. (2010). A new model of random regret minimization. *European Journal of Transport and Infrastructure Research*, 10(2), 181-196.
- Chorus, C. G. (2012). Random regret-based discrete choice modeling a tutorial. SpringerBriefs in business. Berlin ; New York, Springer,: 1 online resource (viii, 55 p.).
- Magidson, J. & Vermunt, J. K. (2007). Removing the scale factor confound in multinomial logit choice models to obtain better estimates of preference. Sawtooth software conference proceedings.
- Van Cranenburgh, S., Guevara, C. A. & Chorus, C. G. (2015). New insights on random regret minimization models. *Transportation Research Part A: Policy and Practice*, 74(0), 91-109.
- Van Cranenburgh, S. & Prato, C. G. (2016). On the robustness of Random Regret Minimization modelling outcomes towards omitted attributes. *Journal of Choice Modelling*, (18), 51-70.
- Van Cranenburgh, S. & Vermunt, J. K. (2015). "Cross-validating Random Regret Minimization results of LatentGOLD Choice." from [www.advancedrrmmodels.com](http://www.advancedrrmmodels.com).
- Vermunt, J. K. & Magidson, J. (2005). Latent GOLD® choice 4.0 user's manual. *Statistical Innovations Inc., Belmont, MA*.