

# Testing if two samples or more described by several variables are significantly different or not

[demoMAHA.xls](#)

## Dataset to test the difference between samples using the Mahalanobis distance

An Excel sheet with both the data and results used in this tutorial can be downloaded by clicking [here](#). This tutorial is based on artificial data that have been generated with the distribution sampling tool of XLSTAT. The first three columns are drawn in a standard normal distribution  $N(0,1)$ . The following three columns have been sampled in a Normal  $N(2, 5)$  distribution for G1, in a Normal  $N(2.2, 5.2)$  distribution for G2 and in a Normal  $N(8, 7)$  distribution for G3.

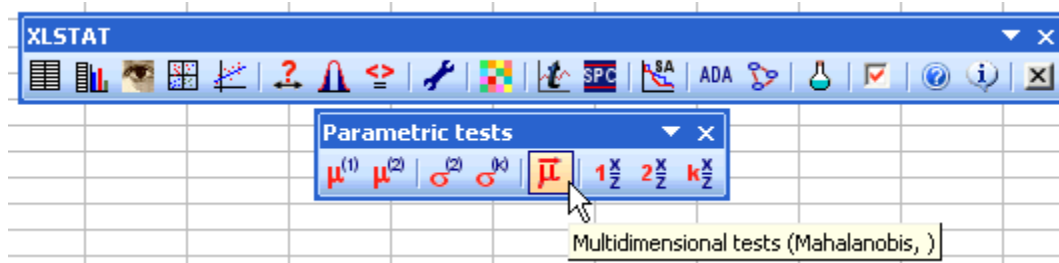
## Testing the difference between samples using the Mahalanobis distance

In order to demonstrate how to use the tool and the relevance of the tests, we will first do a multidimensional test on the first 3 columns, and then on the following 3, and then on the 6 columns together.

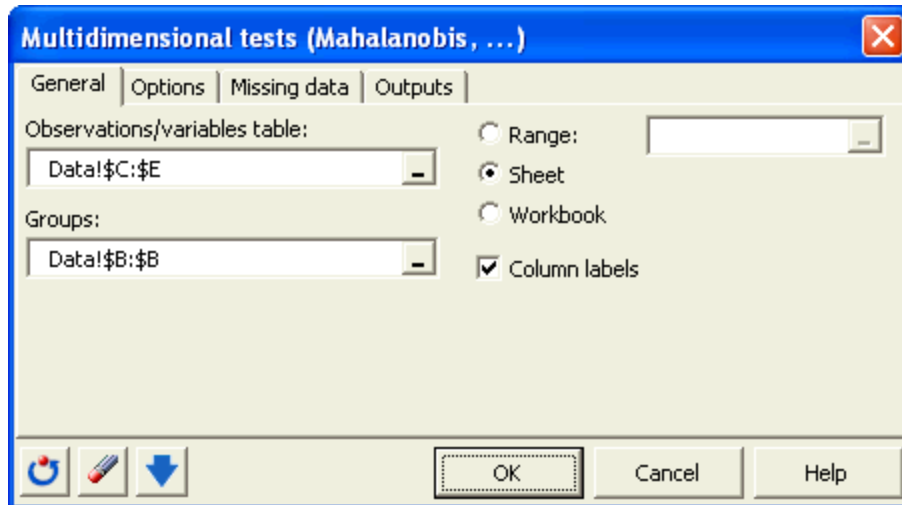
### 1. Tests on the first three columns

#### Setting up a multidimensional test

Once XLSTAT-Pro is activated, select the **XLSTAT / Parametric tests / Multidimensional tests** command, or click on the corresponding button of the **Parametric tests** toolbar (see below).



When you click on the button, a dialog box appears. Select the data corresponding to the first three columns on the Excel sheet, then select column B that contains the group identifiers.



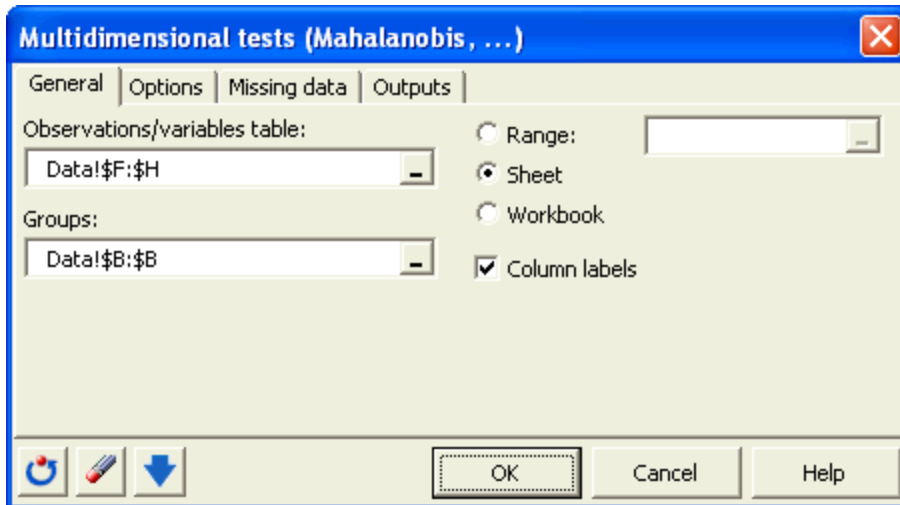
Click **OK** to launch the computations.

### **Results of the multidimensional test**

The results indicate that for both averages (Wilks test) and covariance matrices (Box and Kullback tests), the three groups can be regarded as identical and from the same population. We note with the Fisher's distances that the distance between G1, on the one hand, and G2 or G3 on the other hand, is greater than the distance between G2 and G3, but still not significant however.

Wilks' Lambda test (Rao's approximation):			
Lambda	0,981		
F (Observed)	0,461		
F (Critical value)	2,130		
DF1	6		
DF2	290		
p-value	0,837		
alpha	0,05		
Test interpretation:			
H0: The means vectors of the 3 classes are equal.			
Ha: At least one of the means vector is different from another.			
As the computed p-value is greater than the significance level $\alpha=0,05$ , one should accept the null hypothesis H0.			
The risk to reject the null hypothesis H0 while it is true is 83,67%.			
Mahalanobis distances:			
	G1	G2	G3
G1	0	0,084	0,084
G2	0,084	0	0,002
G3	0,084	0,002	0
Fisher distances:			
	G1	G2	G3
G1	0	0,691	0,686
G2	0,691	0	0,013
G3	0,686	0,013	0
p-values for Fisher distances:			
	G1	G2	G3
G1	1	0,559	0,562
G2	0,559	1	0,998
G3	0,562	0,998	1
<i>The values displayed in bold are significant at the level <math>\alpha=0,01</math></i>			





In this case, tests on averages identify the difference: the test of the Wilks' Lambda concludes that there is a significant difference between the groups means. We notice that the Mahalanobis distances are only meaningful when the group 3 is concerned. It is not surprising that the small difference between the first 2 groups has not been detected as significant, as the group size is too small to identify such a small difference.

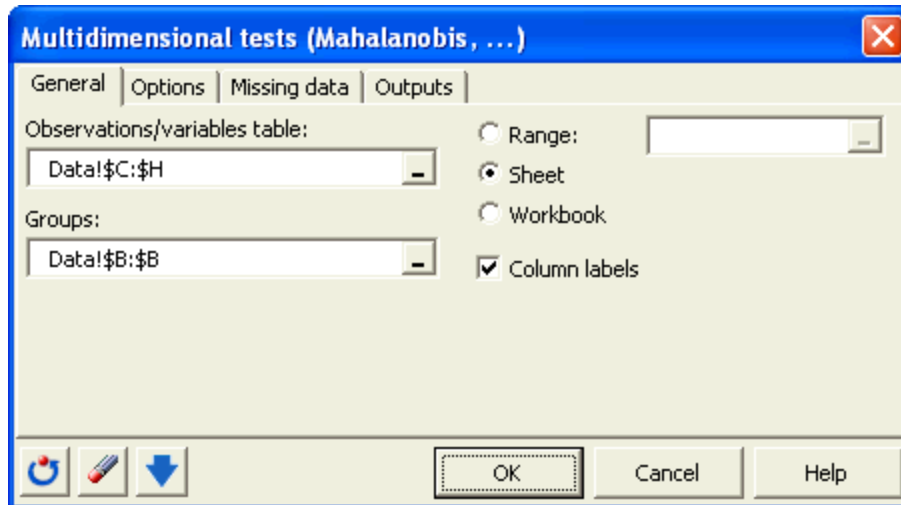
Wilks' Lambda test (Rao's approximation):			
Lambda	0,592		
F (Observed)	14,461		
F (Critical value)	2,130		
DF1	6		
DF2	290		
p-value	< 0,0001		
alpha	0,05		
Test interpretation:			
H0: The means vectors of the 3 classes are equal.			
Ha: At least one of the means vector is different from another.			
As the computed p-value is lower than the significance level alpha=0,05, one should reject the null hypothesis H0, and accept the alternative hypothesis Ha.			
The risk to reject the null hypothesis H0 while it is true is lower than 0,01%.			
Mahalanobis distances:			
	G1	G2	G3
G1	0	0,003	3,134
G2	0,003	0	2,930
G3	3,134	2,930	0
Fisher distances:			
	G1	G2	G3
G1	0	0,028	25,761
G2	0,028	0	24,084
G3	25,761	24,084	0
p-values for Fisher distances:			
	G1	G2	G3
G1	1	0,994	<b>&lt; 0,0001</b>
G2	0,994	1	<b>&lt; 0,0001</b>
G3	<b>&lt; 0,0001</b>	<b>&lt; 0,0001</b>	1
<i>The values displayed in bold are significant at the level alpha=0,01%</i>			

Regarding the covariance matrices, the Box tests are on the borderline of finding a difference, the p-value being equal to 0.06. But the test of Kullback fails to identify the difference. This is due to the size of the groups that are too small to distinguish groups for which variances are  $5^{2^2}$  and  $7^{2^2}$ .

Box test (Chi-square asymptotic approximation):	
-2Log(L)	20,977
Chi-square	20,358
Chi-square	21,026
DF	12
p-value	0,061
alpha	0,05
Test interpretation:	
H0: The within-class covariance matrices are equal.	
Ha: The within-class covariance matrices are different.	
As the computed p-value is greater than the significance level alpha=0,05, one should accept the null hypothesis H0.	
The risk to reject the null hypothesis H0 while it is true is 6,06%.	
Box test (Fisher's F asymptotic approximation):	
-2Log(L)	20,977
F (Observed)	1,696
F (Critical value)	1,752
DF1	12
DF2	104721
p-value	0,061
alpha	0,05
Test interpretation:	
H0: The within-class covariance matrices are equal.	
Ha: The within-class covariance matrices are different.	
As the computed p-value is greater than the significance level alpha=0,05, one should accept the null hypothesis H0.	
The risk to reject the null hypothesis H0 while it is true is 6,07%.	
Kullback's test:	
K (Observed)	10,488
K (Critical value)	21,026
DF	12
p-value	0,573
alpha	0,05
Test interpretation:	
H0: The within-class covariance matrices are equal.	
Ha: The within-class covariance matrices are different.	
As the computed p-value is greater than the significance level alpha=0,05, one should accept the null hypothesis H0.	
The risk to reject the null hypothesis H0 while it is true is 57,32%.	

### 3. Tests on the six columns

This time all columns are selected. In the "Outputs" tab we request correlation and covariance matrices.



Tests on the means yield results very close to the case 2 (see above). The difference between G1 and G2 based on the Mahalanobis distance is slightly lower.

Wilks' Lambda test (Rao's approximation):			
Lambda	0,571		
F (Observed)	7,658		
F (Critical v)	1,786		
DF1	12		
DF2	284		
p-value	< 0,0001		
alpha	0,05		
Test interpretation:			
H0: The means vectors of the 3 classes are equal.			
Ha: At least one of the means vector is different from another.			
As the computed p-value is lower than the significance level alpha=0,05, one should reject the null hypothesis H0, and accept the alternative hypothesis Ha.			
The risk to reject the null hypothesis H0 while it is true is lower than 0,01%.			
Mahalanobis distances:			
	G1	G2	G3
G1	0	0,095	3,305
G2	0,095	0	3,132
G3	3,305	3,132	0
Fisher distances:			
	G1	G2	G3
G1	0	0,380	13,303
G2	0,380	0	12,605
G3	13,303	12,605	0
p-values for Fisher distances:			
	G1	G2	G3
G1	1	0,891	< <b>0,0001</b>
G2	0,891	1	< <b>0,0001</b>
G3	< <b>0,0001</b>	< <b>0,0001</b>	1
<i>The values displayed in bold are significant at the level alpha=0,01?</i>			

However, the tests on the covariance matrices are surprisingly different. Small differences observed on the first 3 columns, and the larger ones observed on the last 3 columns accumulate. Furthermore there are non neglectable covariances between RV1 and RV4, between RV2 and RV5 and between RV3 and RV6. In the end this gives significant differences when the tests are performed on the 6 columns.

Box test (Chi-square asymptotic approximation):	
-2Log(M)	5174,150
Chi-square	4875,803
Chi-square	58,124
DF	42
p-value	< 0,0001
alpha	0,05
Test interpretation:	
H0: The within-class covariance matrices are equal.	
Ha: The within-class covariance matrices are different.	
As the computed p-value is lower than the significance level alpha=0,05, one should reject the null hypothesis H0, and accept the alternative hypothesis Ha.	
The risk to reject the null hypothesis H0 while it is true is lower than 0,01%.	
Box test (Fisher's F asymptotic approximation):	
-2Log(M)	5174,150
F (Observed)	116,010
F (Critical v)	1,384
DF1	42
DF2	64153
p-value	< 0,0001
alpha	0,05
Test interpretation:	
H0: The within-class covariance matrices are equal.	
Ha: The within-class covariance matrices are different.	
As the computed p-value is lower than the significance level alpha=0,05, one should reject the null hypothesis H0, and accept the alternative hypothesis Ha.	
The risk to reject the null hypothesis H0 while it is true is lower than 0,01%.	
Kullback's test:	
K (Observed)	2587,075
K (Critical v)	58,124
DF	42
p-value	< 0,0001
alpha	0,05
Test interpretation:	
H0: The within-class covariance matrices are equal.	
Ha: The within-class covariance matrices are different.	
As the computed p-value is lower than the significance level alpha=0,05, one should reject the null hypothesis H0, and accept the alternative hypothesis Ha.	
The risk to reject the null hypothesis H0 while it is true is lower than 0,01%.	